CONSOB
COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

The quant regulation made easy: an investor-friendly horizontal approach to risk disclosure for all non-equity investment products

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OPINIONS EXPRESSED ARE ONLY OF THE AUTHOR

Syllabus

- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon

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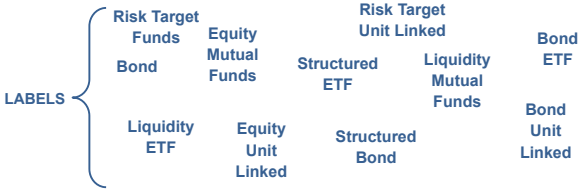
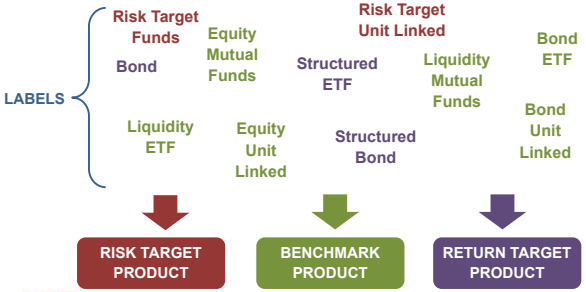
Preliminaries

Non-equity Investment products are currently classified according to the **"labels"** assigned by the issuer or by the regulatory framework.



Preliminaries

Non-equity Investment products **should be classified according to their financial characteristics** and not by **"l**abels**"** assigned by the issuer or by the regulatory framework.



Preliminaries

Consob transparency regulation on the risk profile of non-equity products is based on synthetic indicators – defined through specific quantitative methods – in order to allow investors to take informed investment decisions.

Traditional narrative description of all possible risks associated with a predefined **"label"**

VS

Synthetic indicators robust, objective and backward verifiable

Preliminaries

The transparency approach which is developing at the level of the European Community, through the revision of the reference Directives (UCITS, Prospectus, MiFID, PRIPs), seems to drift again towards a logic based on form (**"label"**) as opposed to substance, as regards the risks which characterize a given product.

Non-simple products, for which an enhanced transparency supervision is viewed as necessary, are identified among different working groups by means of terms which often display a lack coherence, e.g.:

ALMOST COMPLEX
COMPLEX
PACKAGED
STRUCTURED
SUPER-COMPLEX

Preliminaries

CONSOB – STRATEGIC PLAN 2010-2012

CONSOB aims at «*promoting an enhancement of the transparence levels on non-equity products, particularly on the most complex ones which often incorporate components of derivative nature (also implicitly) linked to market and/or credit risk, on the basis of the so-called "three pillars approach"*» beyond a narrative approach.

The *risk-based* transparency approach adopted by CONSOB, by privileging substance over form (**"labels"**) when dealing with risks, represents an opportunity also for issuers, which can take advantage of the best opportunities in the market (even though complex in their structure) in order to offer added value to investors.

Preliminaries

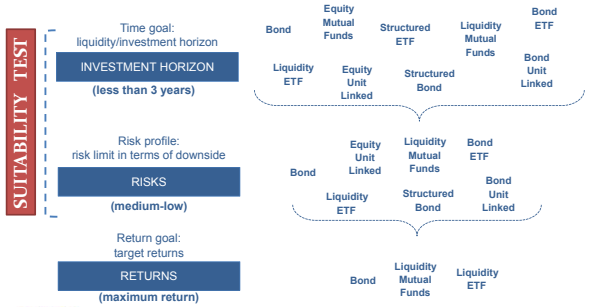
RETURNS/COSTS | RISKS | INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach.

1st Pillar	2nd Pillar	3rd Pillar
Unbundling and Probabilistic performance scenarios	Synthetic risk indicator	The recommended Investment horizon

Preliminaries

These metrics provide a guide to investors in the interpretation of complex information conveyed in the offering document, supporting the decision process by means of a sequential filtering procedure:



Preliminaries

The **UCITS IV Directive** (completely revised) has adopted in the **KID** (document containing the key Investor Information) **only one of the three** indicators promoted by Consob approach (degree of risk), even though with a different specification.

The other two indicators of the *risk-based* approach (*unbundling*/probabilistic scenarios and time horizon) do not find a direct match. In particular:

- CESR has proposed the use of deterministic approaches of the *what-if* kind, in order to implement *performance scenarios*, despite much perplexity has been raised about them.
  In this regard, consider the shared opinion of renowned academics and consumers associations joined together, who have expressed in an open letter to the European Commission extensive criticism about the implementation of *what-if* methodology in KID:
  http://www.crusoe.it/mercato-regole/risk-disclosure-and-the-protection-of-retail-investors-an-open-letter-to-the-cesr-and-the-european/627/
- the recommended time horizon represents a piece of information which the issuer is free to provide on a discretionary basis.

Preliminaries

Recent EC works about **PRIPs** have highlighted, among other things, the following main orientations (even though not definitive for the lack of a shared vision) about pre-contractual information:

- the principle of comparability has been reaffirmed;
- the KID must be used as a reference
  (for those PRIPs characterized by a given maturity date, the information provided through the synthetic risk indicator and the narrative description could be supplemented by an additional indicator related to the time horizon);
- there exists the opportunity of including information about the expected performance of the PRIP (an issue which raises the concerns of many subjects about the fact that introducing performance scenarios could confound investors).

Preliminaries

At the EC level, the debate about the employment of quantitative metrics as opposed to a narrative description is still open.

Numerous countries and associations have taken part in the PRIPs discussion with works of various nature (regulatory and not):

- **Portugal** supports an approach of quantitative kind based on multiple partitions of the probability distribution of expected payoffs;
- **UK insurer AVIVA** proposes also stochastic projections instead of an indicator based on historical volatility;
- **Spain** harshly criticizes the idea of extrapolating UCITS methodology, unable to detect counterparty and volatility risks, to other products;
- **Movement for Risk Transparency** is in favor of the use of forward-looking simulations and probability to disclose the risks and the potential returns of the product and explicitly supports the present Consob approach;
- **UK Association of Investment Companies** considers the KID risk indicator potentially misleading as based on historical volatility;

Reference – Risk Magazine, March 15, 2011 available at http://www.risk.net/risk-magazine/news/2034251/opposition-grows-prips-risk-indicator-proposal).

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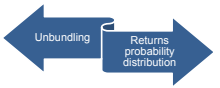
1st Pillar: Unbundling and Probabilistic performance scenarios

RISK TARGET PRODUCT | **BENCHMARK PRODUCT** | **RETURN TARGET PRODUCT**

In "return target" products (e.g. corporate bonds) the connection between the pricing at time zero and the pricing at maturity is evident, as the probability table is a necessary step to obtain the unbundling of the price of the product at time 0.

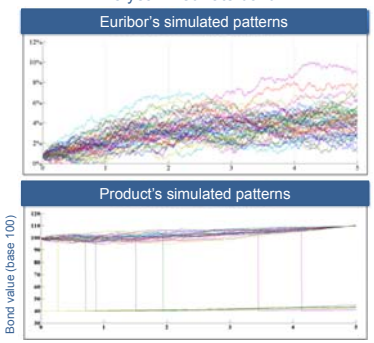
Fair Value
Pricing at time zero



Possible Outcomes
Pricing at maturity

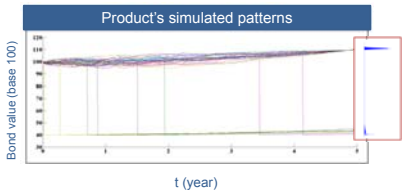
1st Pillar: Unbundling and Probabilistic performance scenarios

5 year fixed-rate bond



1st Pillar: Unbundling and Probabilistic performance scenarios

The final values of the bond at the end of the 5th year provide the probability distribution of potential returns (so-called *pricing* at maturity).



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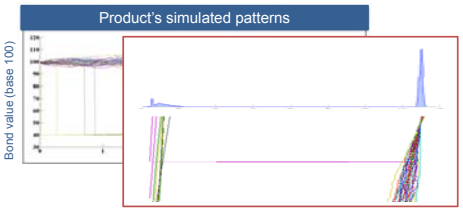
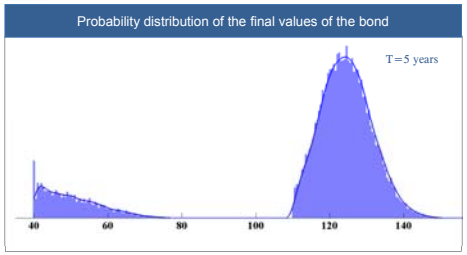


Possible Outcomes
Pricing at maturity

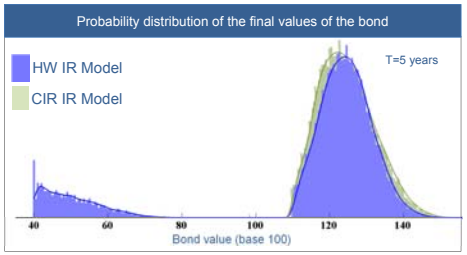
1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: The informative content of the entire probability distribution is very complex to handle for the average retail investor.



1st Pillar: Unbundling and Probabilistic performance scenarios

MODEL RISK: The shape of the probability distribution of potential returns is obviously dependent from the model's assumption.

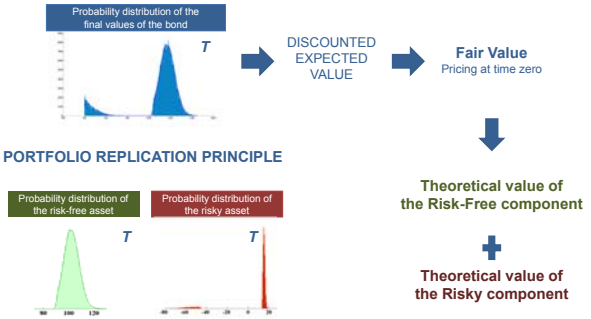


1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: STANDARD SOLUTION

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)



1st Pillar: Unbundling and Probabilistic performance scenarios

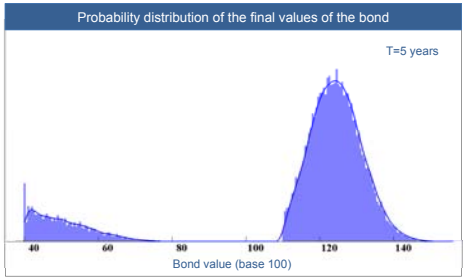
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)

Financial investment table
(Unbundling)

A	Theoretical value of the Risk-Free component	…
B	Theoretical value of the Risky component	…
C = A + B	*Fair value*	…
D	Explicit costs	…
E	Implicit costs	…
F = C + D + E	Issue price	100

1st Pillar: Unbundling and Probabilistic performance scenarios

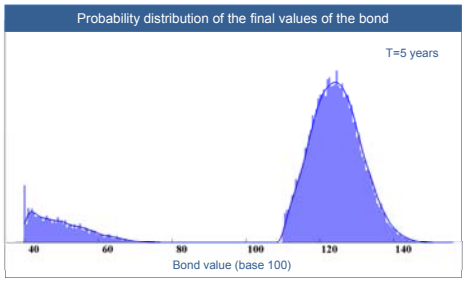
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



It's interesting to explore a different representation of the information contained in the probability distribution which could be useful for the average investor

1st Pillar: Unbundling and Probabilistic performance scenarios

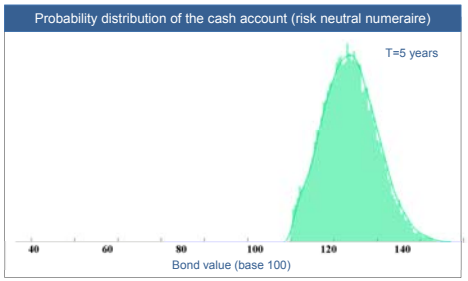
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



In order to provide the investor with a representation fair, easy to understand and resilient to the model's risk, a simple rescaling with respect to the risk-neutral measure numeraire is presented

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



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1st Pillar: Unbundling and Probabilistic performance scenarios

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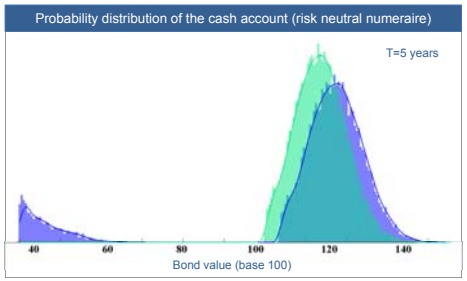
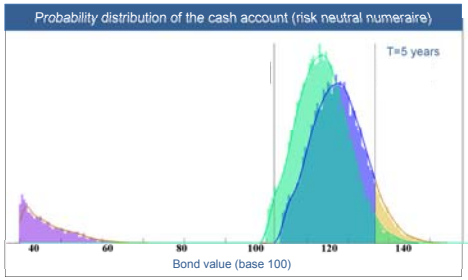


The superimposition of the product's probability distribution with the cash account naturally defines three different events which are effectively meaningful for the investor.

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



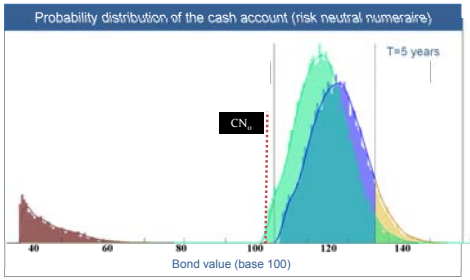
The performance is lower than the risk-free asset

The performance is in line with the risk-free asset

The performance is higher than the risk-free asset

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



The performance is negative

The performance is positive and lower than the risk-free asset

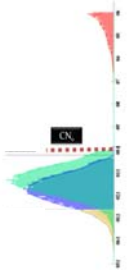
The performance is positive and in line with the risk-free asset

The performance is positive and higher than the risk-free asset

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)

Probabilistic performance scenario table



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1) e (2)

Connection between the pricing at time zero and the pricing at the end of recommended investment horizon

Time Zero

Financial investment table

End of the recommended investment horizon

Table of probabilistic performance scenarios

1:1 Relationship

Example

Zeta Bank BOND

DESCRIPTION	Four-year contingent convertible bonds that provides the mandatory conversion into shares of the issuer in predefined date and pricing conditions according to a basket of put and call of European and American options.
STRUCTURE	RETURN TARGET
1st PILLAR	

***Unbundling* Table**

Theoretical value of the Risk-Free component	
Theoretical value of the Risky component	
Theoretical value of the product	95.17
Costs	4.83
Issue price	100.00

PROBABILISTIC SCENARIOS	Event Probability	Mean value
The performance is negative	68.50%	59.2
The performance is positive but lower than the risk-free asset	2.80%	103,8
The performance is positive and in line with the risk-free asset	4.40%	113,7
The performance is positive and higher than the risk-free asset	24.30%	162.3

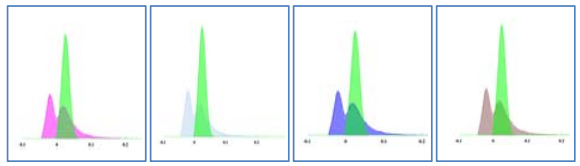
1st Pillar: Unbundling and Probabilistic performance scenarios

MODEL RISK: CONSOB REGULATION

The model risk arising from the right to freely use the proprietary models is solved with the reduction in granularity of events

Many possible choices…

Heston | Merton | V G | NIG



1st Pillar: Unbundling and Probabilistic performance scenarios

MODEL RISK: CONSOB REGULATION

The results of the various models show differences between each box of less than 5%

… the following output is obtained:

Heston | Merton | V G | NIG

1st Pillar: Unbundling and Probabilistic performance scenarios

|Δ| < 4.7%

Heston | Merton | V G | NIG

1st Pillar: Unbundling and Probabilistic performance scenarios

|Δ| < 2.7%

Heston | Merton | V G | NIG

|A| < 3.7%

|A| < 1.2%

Probabilistic Performance Scenarios **vs** **What-if**

Probabilistic Performance Scenarios **vs** **What-if**

Example:

Narrative description of the product's features.

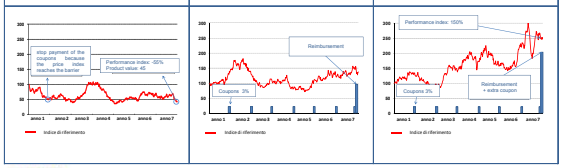
The structured product, whose maturity is **7 years**, presents returns which are linked to the **Dow Jones Eurostoxx** Index.

The fund gives annual coupons, equal to 3% of the initial invested capital, but:

- if, at any time in the fund life, the reference index falls below 50% of its initial value:
  - the payment of coupons is interrupted;
  - at the end of the 7th year the fund will pay back the value of the initial invested capital increased or reduced on the basis of the index *performance*;
- if the index never falls below 50% of its initial value, at the end of the 7th year the fund will pay:
  - the initial value of the investment;
  - moreover, if at the maturity date the index value is greater or equal to twice its initial value, the fund will pay an additional coupon equal to the initial value of the investment.

Probabilistic Performance Scenarios **vs** **What-if**

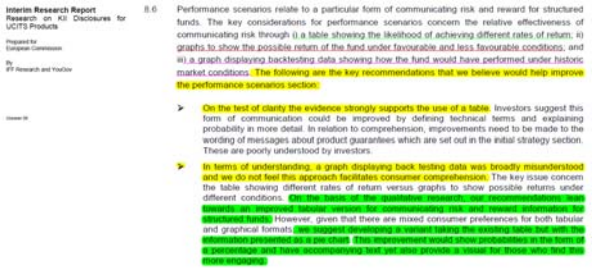
"What-if" representation		
Unfavourable scenario	**Neutral scenario**	**Favourable scenario**
The Dow Jones Eurostoxx value falls below 50% during the first year of the fund's life and at the end of the 7th year the performance of the Dow Jones Eurostoxx index is equal to 55%. The fund does not pay any coupon and at maturity, it pays 45 on an initial investment of 100.	The Dow Jones Eurostoxx value never falls below 50% during the life of the fund and at the end of the 7th year the value of the Dow Jones Eurostoxx index is less than twice its initial value. The fund pays every year a 3% coupon and at maturity it pays the initial value of the investment.	The Dow Jones Eurostoxx value never falls below 50% during the life of the fund and at the end of the 7th year the performance of the Dow Jones Eurostoxx index is equal to 130%. The fund pays every year a 3% coupon and at maturity it pays twice the initial value of the investment .



Probabilistic Performance Scenarios **vs** **What-if**

Representation through the probabilistic performance scenarios table at the end of the 7th year			
SCENARIOS	**PROBABILITY**	**MEAN VALUE**	**YIELD**
The performance is negative	38.71%	55.52	-8.06%
The performance is positive but lower than the risk-free asset	8.45%	110.58	1.45%
The performance is positive and in line with the risk-free asset	36.09%	123.13	3.02%
The performance is positive and higher than the risk-free asset	16.75%	223.27	12.16%

Probabilistic Performance Scenarios **vs** ***What-if***



Interim Research Report Research on KII Disclosures for UCITS Products

8.6 Performance scenarios relate to a particular form of communicating risk and reward for structured funds. The key considerations for performance scenarios concern the relative effectiveness of communicating risk through i) a table showing the likelihood of achieving different rates of return; ii) graphs to show the possible return of the fund under favourable and less favourable conditions; and iii) a graph displaying backtesting data showing how the fund would have performed under historic market conditions. The following are the key recommendations that we believe would help improve the performance scenarios section:

- On the test of clarity the evidence strongly supports the use of a table. Investors suggest this form of communication could be improved by defining technical terms and explaining probability in more detail. In relation to comprehension, improvements need to be made to the wording of messages about product guarantees which are set out in the initial strategy section. These are poorly understood by investors.
- In terms of understanding, a graph displaying back testing data was broadly misunderstood and we do not feel this approach facilitates consumer comprehension. The key issue concern the table showing different rates of return versus graphs to show possible returns under different conditions. On the basis of the qualitative research, we recommend moving from towards an improved tabular version for communicating risk and reward information for structured funds. However, given that there are mixed consumer preferences for both tabular and graphical formats we suggest developing a variant taking the existing table but with the information presented as a pie chart. This improvement would show probabilities in the form of a percentage and have accompanying text yet also provide a visual for those who find this more engaging.

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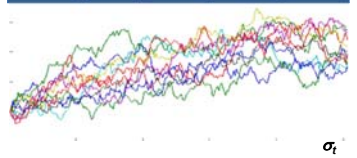
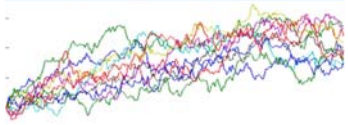
Volatility of the product's potential returns



Volatility is the most immediate risk measure and it has a one-to-one relationship with whatever loss measure (VaR, ES, etc.)

2nd Pillar: Synthetic risk indicator

Volatility of the product's potential returns



σ_t

VOLATILITY

e.g. : geometric brownian motion

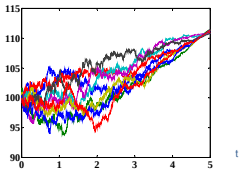
$$dS_t = rS_t dt + \sigma S_t dW_t$$

$$VaR_{\alpha,1year} = e^{\sigma\,\Phi^{-1}(\alpha)\sqrt{\frac{252}{252}}+\left(r-\frac{\sigma^2}{2}\right)\frac{252}{252}} - 1$$

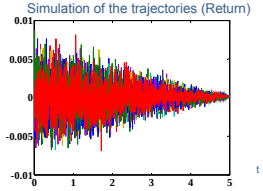
$$ES_{\alpha,1year} = \frac{1}{\alpha} e^{r\frac{252}{252}}\,\Phi\left(\Phi^{-1}(\alpha) - \sigma\sqrt{\frac{252}{252}}\right) - 1$$

2nd Pillar: Synthetic risk indicator

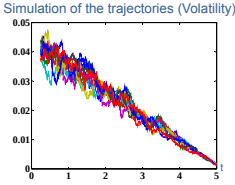
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



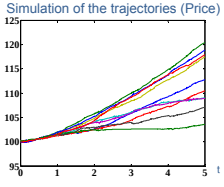
Simulation of the trajectories (Volatility)



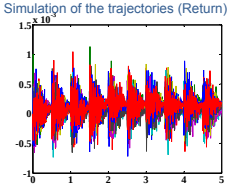
Non-*equity product*: Fixed bond like

2nd Pillar: Synthetic risk indicator

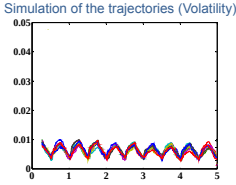
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



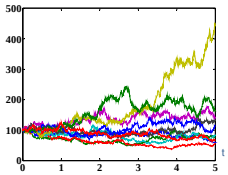
Simulation of the trajectories (Volatility)



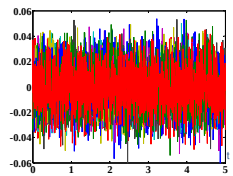
Non-*equity product*: Floater bond like

2nd Pillar: Synthetic risk indicator

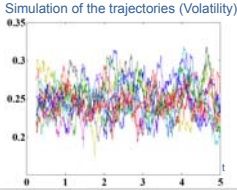
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



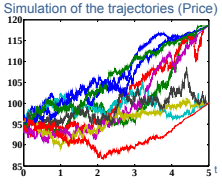
Simulation of the trajectories (Volatility)



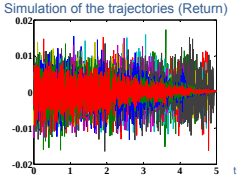
Non-*equity product*: Equity like

2nd Pillar: Synthetic risk indicator

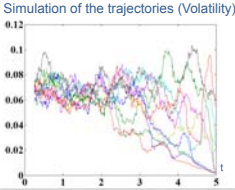
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



Simulation of the trajectories (Volatility)

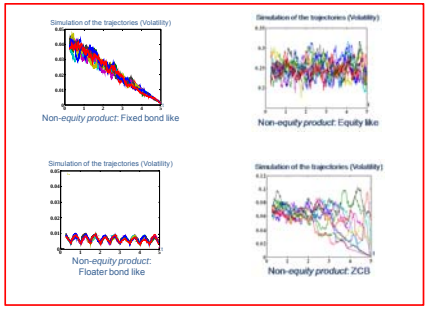


Non-*equity product*: ZCB

2nd Pillar: Synthetic risk indicator

COMPLEXITY FOR RETAIL INVESTORS

The volatility patterns are abstract objects that an average investor cannot handle.



2nd Pillar: Synthetic risk indicator

Conversely, a table with qualitative labels that characterizes the risk classes is very easy to understand

Risk Classes
Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

Accordingly, a quantitative criterion that maps coherently any volatility interval into a corresponding qualitative risk class is needed

2nd Pillar: Synthetic risk indicator

Volatility of the product's potential returns

DEGREE OF RISK

MEASUREMENT: product's positioning inside a grid of *n* volatility intervals

REPRESENTATION: mapping of any volatility interval into a corresponding qualitative risk class

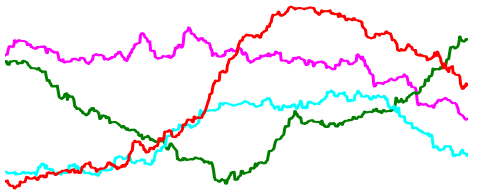
RISK

Risk Classes	Volatility Intervals
Very Low	$\sigma_{1,min}$ $\sigma_{1,max}$
Low	$\sigma_{2,min}$ $\sigma_{2,max}$
Medium-Low	$\sigma_{3,min}$ $\sigma_{3,max}$
Medium	$\sigma_{4,min}$ $\sigma_{4,max}$
Medium-High	$\sigma_{5,min}$ $\sigma_{5,max}$
High	$\sigma_{6,min}$ $\sigma_{6,max}$
Very High	$\sigma_{7,min}$ $\sigma_{7,max}$

2nd Pillar: Synthetic risk indicator

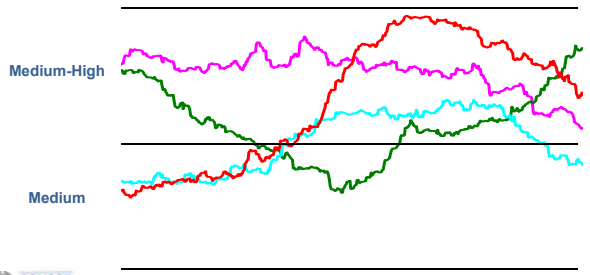
Products with the same risk budget

must have the same degree of risk



σ_t

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations



Volatility intervals have to be suitably calibrated
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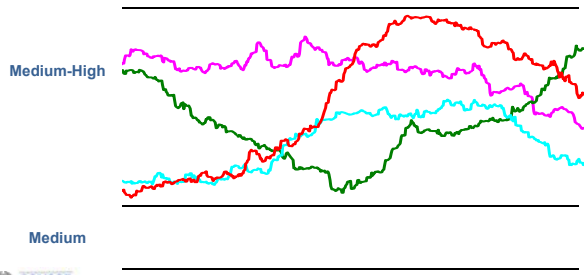
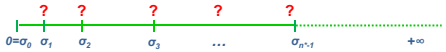


Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

THE ISSUE

Defining suitable requirements to partition the volatility space $[0,+\infty)$ into an optimal number n^* of subsequent intervals with optima extrema



Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

Requirement n.1
the **optimal grid** of volatility intervals
has to be **consistent** with the **principle**:

+ RISK + LOSSES

VOLATILITY INTERVALS MUST HAVE
AN INCREASING WIDTH IN ABSOLUTE TERMS

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

Requirement n.2
the optimal grid of volatility intervals must be
market feasible

REALIZED VOLATILITY CONSISTENT WITH MARKET EXPECTATION OF FUTURE VOLATILITY
(UNLESS FOR SIGNIFICANT SUDDEN SHOCKS)

Realized volatility

Any product on the markets reflects specific asset management policies

Historical data can be "dirty"

need for comparability

1st INTUITION

It has to be studied a theoretical product
managed by an automatic asset manager
who has a specific risk budget,
identified by a given volatility interval

Market expectations of future volatility

future volatility is predicted by exploiting
information embedded in recently observed data

2nd INTUITION

Market expectation is given by volatility prediction intervals based on proper diffusive models

Assessing ***market feasibility***

putting together the two ingredients

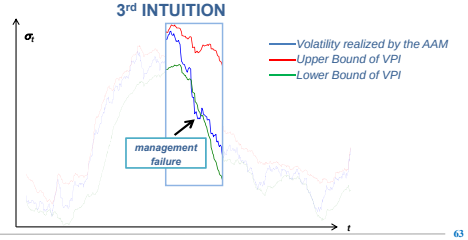
3rd INTUITION

It requires to study when the volatility realized by the automatic asset manager is outside the volatility prediction interval
(so-called *management failures*)

Assessing ***market feasibility***

putting together the two ingredients

3rd INTUITION



2nd Pillar: Synthetic risk indicator

The three intuitions lead to restate the requirement of ***market feasibility*** as a ***two-fold problem***

NOT ABNORMALITY (for any interval):

none interval displays an **abnormal** number of management failures

SUITABLE WIDTH OF THE INTERVAL

HOMOGENEITY (across risk budgets):

the number of management failures is (almost) **the same** for all volatility intervals

NO INCENTIVES TO CHOOSE ANY SPECIFIC RISK BUDGET

2nd Pillar: Synthetic risk indicator

Solving for the optimal grid

ON THE FULL VOLATILITY SPACE [0, +∞)

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

The **optimal grid** of volatility intervals is **consistent** with the **1st requirement**:

+ RISK + LOSSES

2nd Pillar: Synthetic risk indicator

CONSOB **vs** **CESR**

Volatility grid

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
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Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

Annualized volatility estimated on **daily** returns over **1 year**

Migration

3 months out of the risk class indicated in the prospectus

Volatility grid

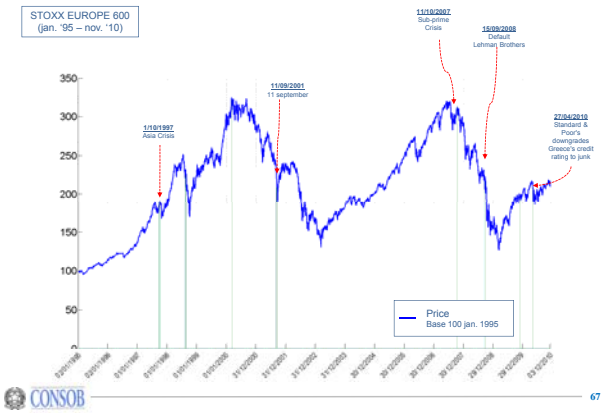
Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0%	0.5%
Low	0.5%	2.0%
Medium-Low	2.0%	5.0%
Medium	5.0%	10.0%
Medium-High	10.0%	15.0%
High	15.0%	25.0%
Very High	25.0%	

Annualized volatility estimated on **weekly** returns over **5 years**

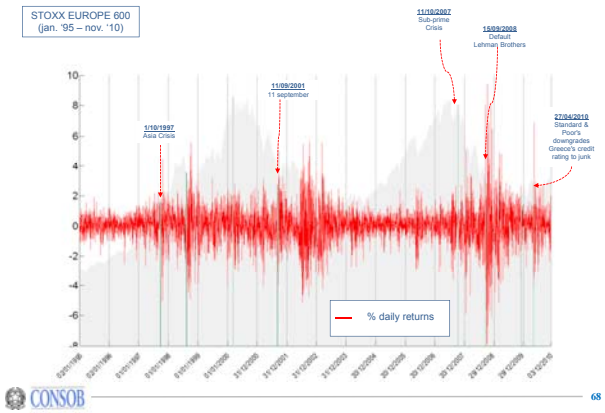
Migration

4 months out of the risk class indicated in the prospectus

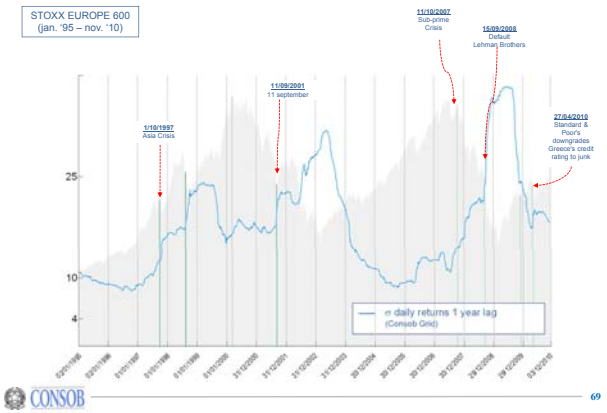
2nd Pillar: Synthetic risk indicator



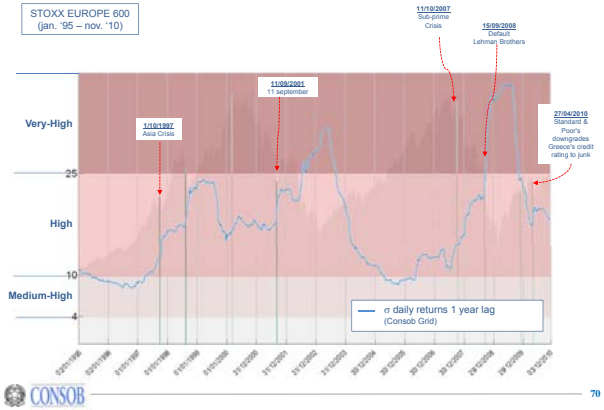
2nd Pillar: Synthetic risk indicator



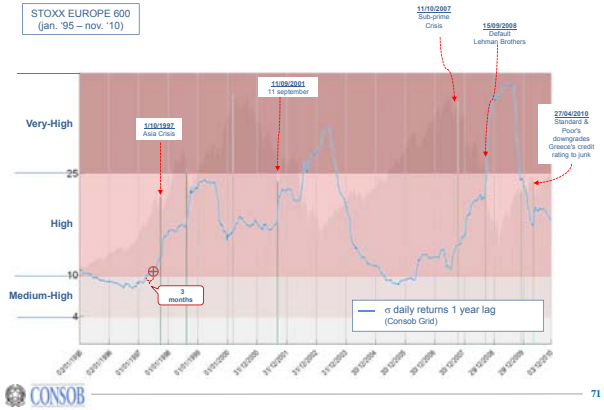
2nd Pillar: Synthetic risk indicator



2nd Pillar: Synthetic risk indicator



2nd Pillar: Synthetic risk indicator



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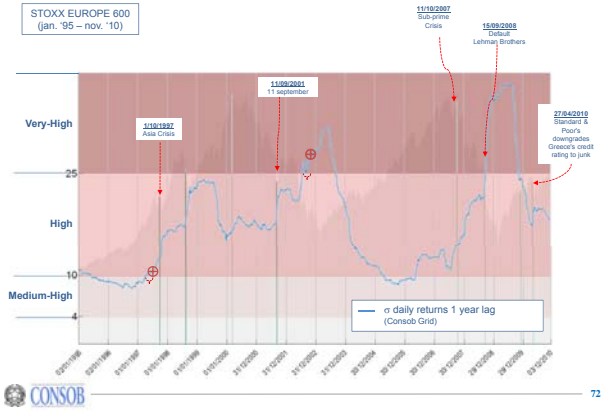

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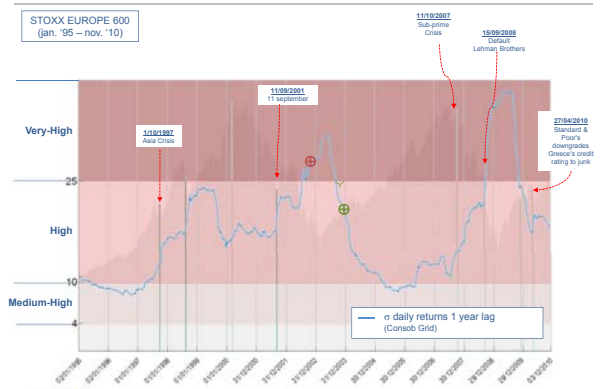

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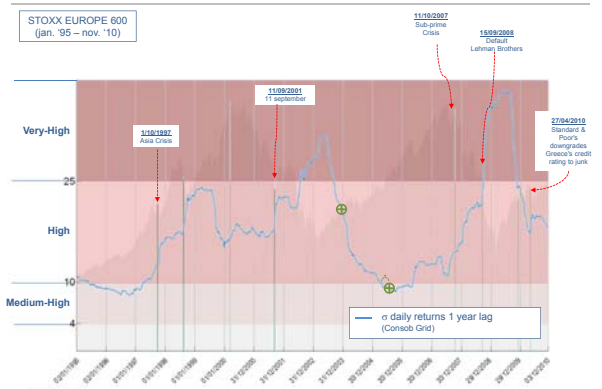

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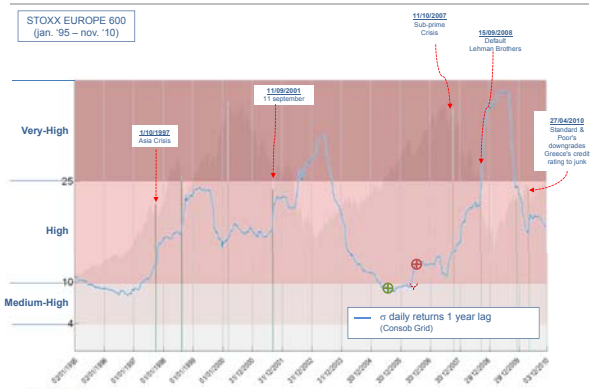
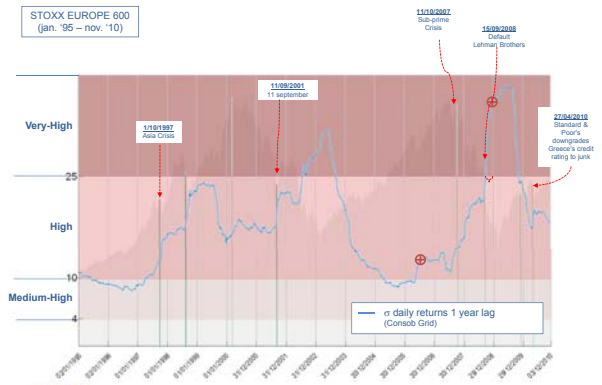


2nd Pillar: Synthetic risk indicator



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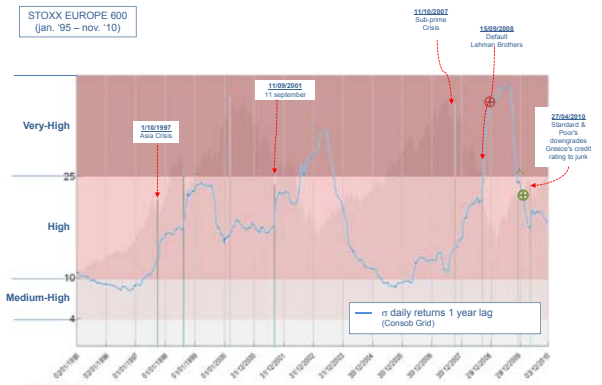

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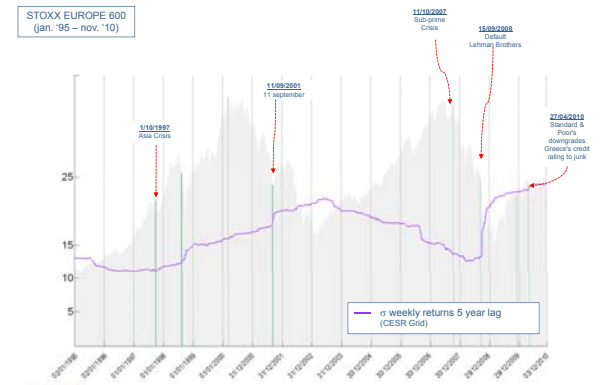

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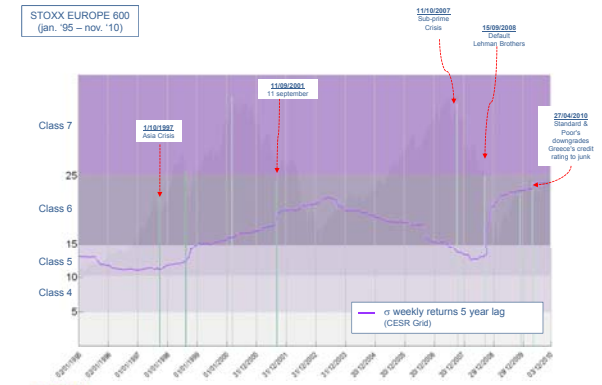

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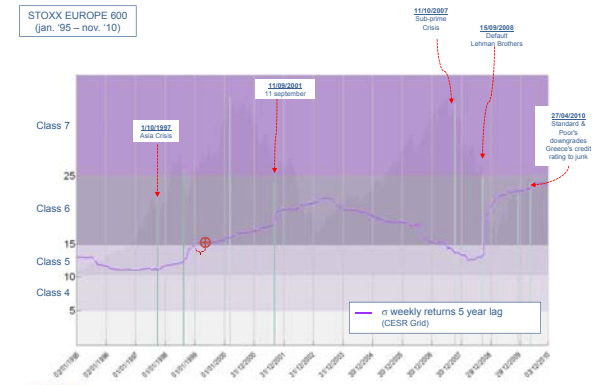

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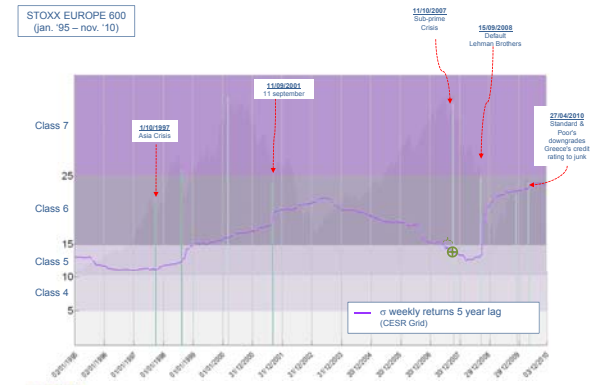

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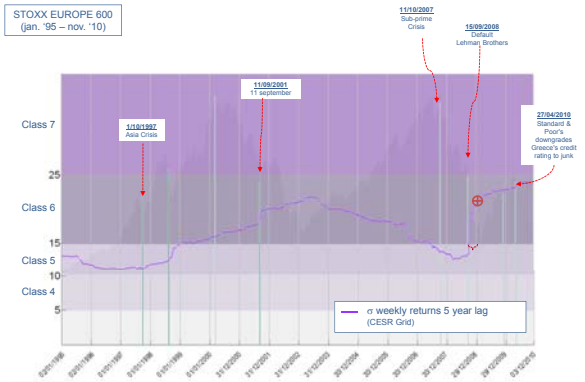


Syllabus

- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon

3rd Pillar : The recommended investment time horizon

RISK TARGET PRODUCT | **BENCHMARK PRODUCT** | **RETURN TARGET PRODUCT**

The recommended investment time horizon

for performance target products the recommended investment horizon is inherent to their financial engineering, as it can be identified as the period of validity (or the time to maturity) of their target

The payoff at maturity implicitly identifies the time horizon

3rd Pillar : The recommended investment time horizon

RISK TARGET PRODUCT | **BENCHMARK PRODUCT** | **RETURN TARGET PRODUCT**

The recommended investment time horizon

The use of solutions aimed at ensuring the liquidity and/or marketability of a return target product influences its recommended investment time horizon

The event to study from a probabilistic point of view is related to possible exit strategies after having recovered all the costs of the product :

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time

The "minimum" recommended investment time horizon

3rd Pillar : The recommended investment time horizon

RISK TARGET PRODUCT | **BENCHMARK PRODUCT** | **RETURN TARGET PRODUCT**

The "minimum" recommended investment time horizon

For risk target products, the natural way to define a cost recovery event is also:

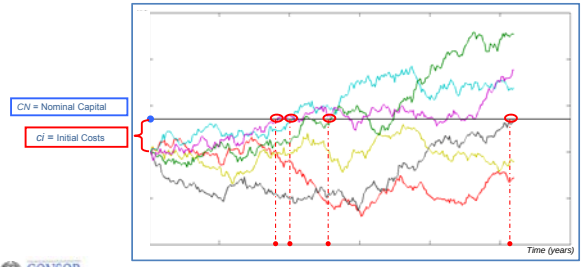
The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time

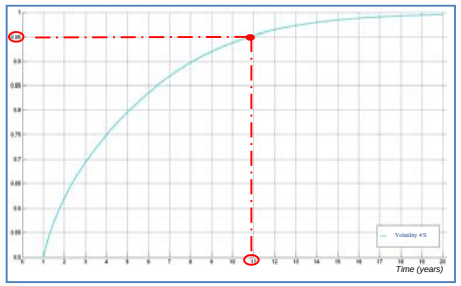
3rd Pillar : The recommended investment time horizon

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



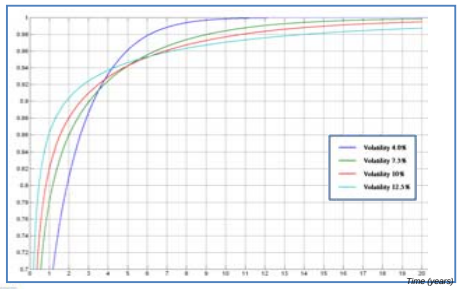
3rd Pillar : The recommended Investment horizon

The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



3rd Pillar : The recommended Investment horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



3rd Pillar : The recommended Investment horizon

Anyway, the recommended **minimum** investment time horizon…

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \le T\right] = \alpha \right\}$$

…. must be coherent with the principle

+ VOLATILITY + TIME HORIZON

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$T \to \infty, dt \to 0$

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{dP}{d\sigma} = \left(-4 \frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

FIRST ORDER ASYMPTOTIC CONDITION

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Since it is safe to assume a positive interest rate *r* in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

In other words, for a given confidence level, as the volatility grows, the recommended investment time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$T \to \infty, dt \to 0$

1. $\begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases}$

$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$

2. ~~$(\bar{r} - cr) \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Summarizing the results of the asymptotic analysis in continuous time:

- As T →∞, for given a confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T^*, beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

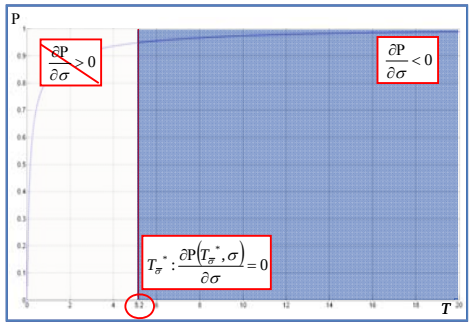
General Framework:

$$\begin{cases} T \to \infty \\ dt \to 0 \\ P(\infty, \sigma) \\ 1. \begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases} \end{cases} \Rightarrow \begin{cases} T \ \ finite \\ dt \to 0 \\ P(T, \sigma) \\ (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T \to \infty} \frac{\partial P(T,\sigma)}{\partial \sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \lim_{T \to \infty} \frac{\partial^2 P(T,\sigma)}{\partial \sigma^2} > 0 \end{cases}$$

Everything shown above also holds with *T* finite!

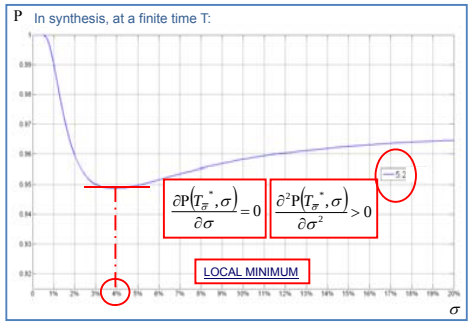
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

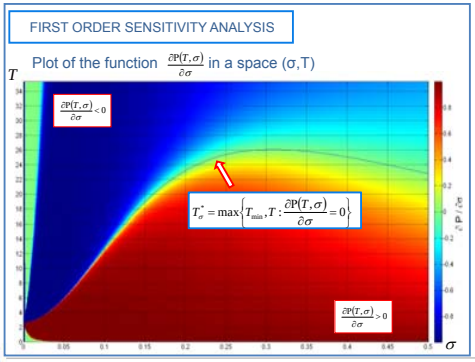


3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

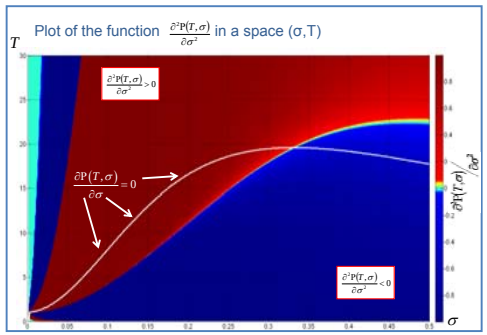
Plot of the function $\frac{\partial P(T,\sigma)}{\partial \sigma}$ in a space (σ,T)



3rd Pillar : The recommended Investment horizon

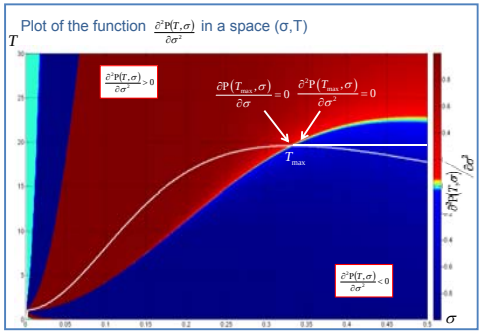
Searching the minimum: **the 2nd order condition**

Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$ in a space (σ,T)



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended investment time horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

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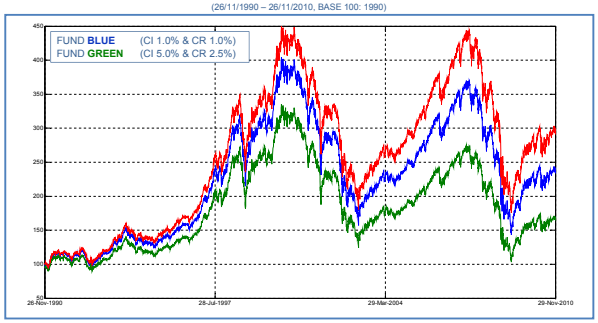
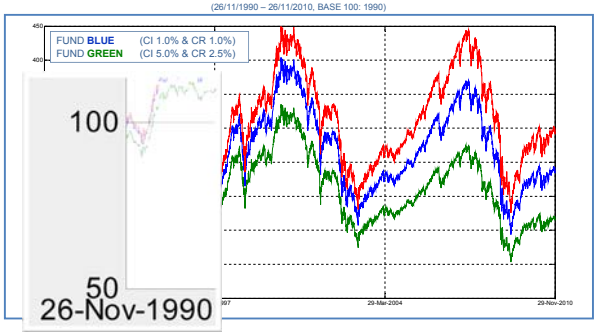


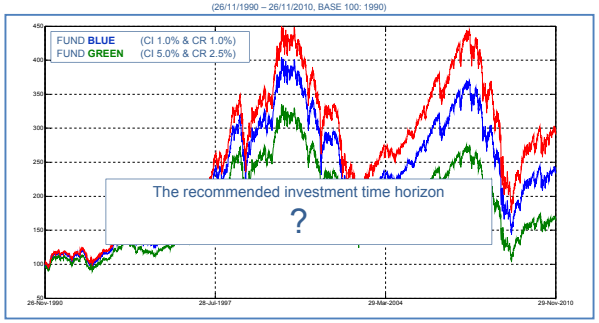
3rd Pillar : The recommended investment time horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



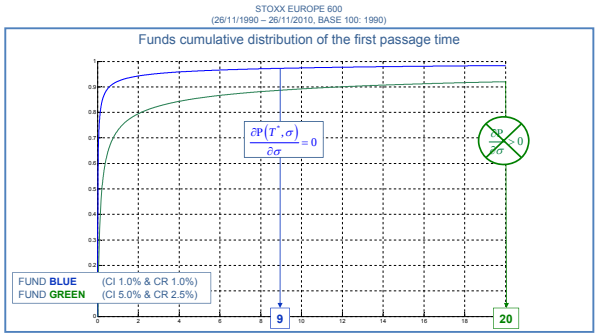
3rd Pillar : The recommended investment time horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended investment time horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



Examples

DERIVATIVE PRODUCT	
DESCRIPTION	The product presents the following *payoff*: • if the reference equity index remains above 50% of its initial value, the investor receives a quarterly fixed coupon equal to 1.8% of the issue price and the payment of the invested capital at maturity; • if the index reaches 50% of its initial value the coupon flow is interrupted and at maturity the investor receives a payment for the investment equal to the *performance* of the index.
1st PILLAR	***Unbundling* Table** Theoretical value of the Risk-Free component: 0.00 Theoretical value of the Risky component: 88.44 Theoretical value of the product: 88.44 Costs: 11.56 Issue price: 100.00 **PROBABILISTIC SCENARIOS** / **Event Probability** / **Mean Value** The performance is negative: 46,160% / 60,120% The performance is positive but lower than the risk-free asset: 4,860% / 107,130% The performance is positive and in line with the risk-free asset: 3,430% / 128,380% The performance is positive and higher than the risk-free asset: 45,550% / 152,820%
2nd PILLAR	Degree of Risk: Medium-High
3rd PILLAR	Recommended investment time horizon: 6 years

Examples

STRUCTURED PRODUCT	
DESCRIPTION	The investor receives fixed coupons with values increasing from 1% to 2.5% for the first 3 years. At maturity, she receives the payment of the issue price possibly increased by an additional bonus equal to 35% of the reference index performance (if positive) multiplied by the issue price.
1st PILLAR	***Unbundling* Table** Theoretical value of the Risk-Free component: 85.62 Theoretical value of the Risky component: 7.09 Theoretical value of the product: 92.71 Costs: 7.29 Issue price: 100.00 **PROBABILISTIC SCENARIOS** / **Event Probability** / **Mean Value** The performance is negative: 8,72% / 45,59% The performance is positive but lower than the risk-free asset: 0% / 0% The performance is positive and in line with the risk-free asset: 87,10% / 111,97% The performance is positive and higher than the risk-free asset: 4,18% / 155,91%
2nd PILLAR	Degree of Risk: Medium
3rd PILLAR	Recommended investment time horizon: 7 years

Examples

SUBORDINATED BOND	
DESCRIPTION	Subordinated bond with a 7 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30% and characterized by an amortizing plan from the 3rd to the 7th year.
STRUCTURE	RETURN TARGET
1st PILLAR	

Unbundling Table

Theoretical value of the Risk-Free component	83.361
Theoretical value of the Risky component	11.032
Theoretical value of the product	94.393
Costs	5.607
Issue price	100.00

PROBABILISTIC SCENARIOS	Event Probability	Mean value
The performance is negative	23,51%	54,73%
The performance is positive but lower than the risk-free asset	0,55%	100,23%
The performance is positive and in line with the risk-free asset	74,48%	133,05%
The performance is positive and higher than the risk-free asset	1,46%	144,66%

2nd PILLAR	Degree of Risk: Medium-High
3rd PILLAR	Recommended investment time horizon: 7 years

Bibliography

- A quantitative risk-based approach to the transparency on non-equity investment products
  www.consob.it *Quaderno di finanza n. 63*
- A Quantitative Framework for Assessing the Risk-Reward Profile of Non-Equity Investment Products
  Riskbooks – *forthcoming*

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The quant regulation made easy: an investor-friendly horizontal approach to risk disclosure for all non-equity investment products

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Head of the Quantitative Analysis Unit
CONSOB

Risk Europe | Brussels 4-7 April 2011

OPINIONS EXPRESSED ARE ONLY OF THE AUTHOR