ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

MatemateS

DIPARTIMENTO DI MATEMATICA PER LE SCIENZE ECONOMICHE E SOCIALI

FRONTIERS IN FINANCIAL MARKETS MATHEMATICS

"Pricing Illiquid Securities"
Summer School Edition 2009
MatemateS
University of Bologna

Bologna, 13-17 July, 2009

The **Summer School in Financial Market Mathematics** wants to provide the audience with the state of the art of advanced topics in financial mathematics by means of lectures from the most outstanding scholars working on frontier issues. The program is spread over a five day period. The first day will provide an introductory tutorial to the topic, the following four will be devoted to in-depth analysis of the issues.

Sixth International Summer School:

Topic: Pricing Illiquid Securities One of the main topics raised by the recent financial crises is the problem of market liquidity. Lack of liquidity in the market is a problem common to risk managers, pricers, auditors and regulators. Particularly interesting issues in this topic are the definition of pricing bounds, the selection of worst case scenarios for liquidity, the definition of axioms for the choice of acceptable prices for sale and purchase of products, the design of alternative trading strategies to gauge the performance of investment in illiquid securities.

Faculty

Ales Cerny, Cass Business School, London
Claudio Tebaldi, Bocconi University, Milan
Marcello Minenna, CONSOB
Giacomo Scandolo, University of Florence
Tiziano Vargiolu, University of Padua
Sabrina Mulinacci, University of Bologna
Umberto Cherubini, University of Bologna

VENUE: DEPARTMENT OF MATHEMATICAL ECONOMICS (MATEMATES), VIALE FILOPANTI 5, 40126 BOLOGNA

Preliminary Program

Monday 13 July, 2009:

h. 10,00-10,30: Registration and Address of the Head of Department
h. 10.30-12-30: ***Liquidity Risk A Tutorial***, Umberto Cherubini, MatematES, Bologna
h. 14.00-16.00: ***Pricing Illiquid Securities: A Tutorial***, Sabrina Mulinacci, MatematES, Bologna

Tuesday 14 July, 2009

h.10,30-12,30: ***Risk Measures***, Giacomo Scandolo, University of Florence
h.14.00-16,00 ***Liquidity Risk Measures***, Giacomo Scandolo, University of Florence

Wednesday 15 July, 2009

h.10,30-12,30: ***A Risk Measure at Work: Shortfall Risk***, Tiziano Vargiolu, University of Padua
h.14.00-16,00 ***Quantitative Measures for a Unified Approach to Risk Disclosure for Financial Products***, Marcello Minenna, Consob

Thursday 16 July, 2009

h.10,30-12,30: ***Mean Variance Hedging with Liquidity Effects***, Ales Cerny, Cass Business School, London
h.14.00-16,00 ***Maximizing Non-Expected Utility***, Ales Cerny, Cass Business School, London

Friday 17 July, 2009

h.10,30-12,30: ***Financial Valuation when Some Assets Are Illiquid***, Claudio Tebaldi, Boccon University, Milan
h.12,00-14,00: ***Uncertainty Aversion and Non-Additive Utility***, Sabrina Mulinacci, Umberto Cherubini, University of Bologna

CONSOB

COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

Quantitative measures for an unified approach to risks disclosure for financial products

FRONTIERS IN FINANCIAL MARKETS MATHEMATICS - BOLOGNA, 13-17 LUGLIO 2009

OUTLINE

Non-equity investment products

Financing products

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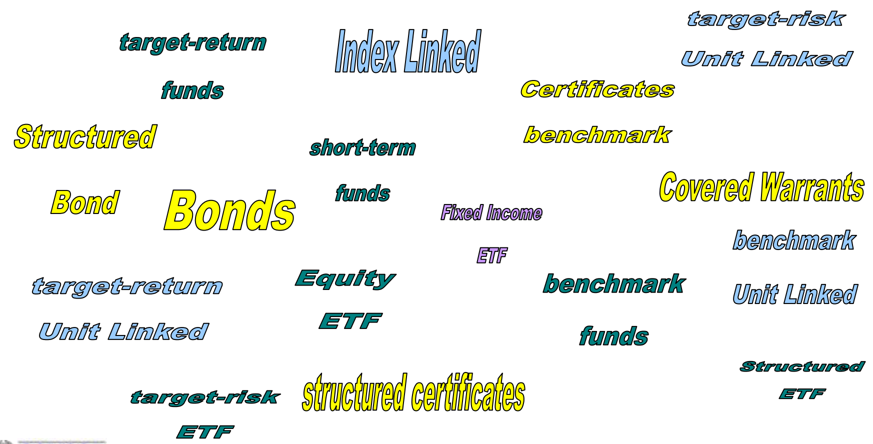
Non-equity investment products

Financing products

The implementation of the disclosure regulation on the risk-profile of non-equity investment products should allow the investor, even assisted by a financial advisor, to choose the financial product more suitable to his investment objectives.



THREE DIFFERENT DIRECTIVES FOR THE SAME FINANCIAL ENGINEERING

UCITS Directive	Prospectus Directive	Life Assurance Directive
Equity, benchmark, ETF, funds, target-risk, ETF, target-return, funds, short-term, funds, Structured ETF	Structured, Bond, Certificates, benchmark, Bonds, Covered Warrants, structured certificates	target-risk, Unit Linked, Index Linked, benchmark, Unit Linked, target-return, Unit Linked

Even if the heterogeneity in names of the products, categories of the issuers, distribution channels and costs applied create the appearance of actual differences also in the underlying financial engineering, the universe of investment products can effectively be classified into the following three types of financial structures:

Risk Target products

risk optimization
vs
return sub-optimization

Return Target products

return optimization
vs
risk sub-optimization

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***Benchmark* products**

investment policy which is anchored to a benchmark, in relation to it the asset management style may be either passive or active

Even if the heterogeneity in names of the products, categories of the issuers, distribution channels and costs applied create the appearance of actual differences also in the underlying financial engineering, the universe of investment products can effectively be classified into the following three types of financial structures:

Risk Target products	*Benchmark* products	Return Target products
target-risk, ETF, target-risk, Unit Linked	benchmark, funds, Certificates, benchmark, Equity, ETF, benchmark, Unit Linked	Structured, Structured ETF, Bond, target-return, funds, Index Linked, structured certificates, Bonds, target-return, Unit Linked, Covered Warrants

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

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Time goal: liquidity/investment horizon

INVESTMENT HORIZON

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal: liquidity/investment horizon

INVESTMENT HORIZON

Risk profile: risk limit in terms of downside

RISKS

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal: liquidity/investment horizon

INVESTMENT HORIZON

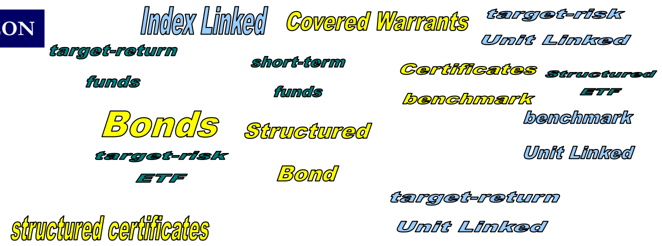
Risk profile: risk limit in terms of downside

RISKS

Return goal: desired returns

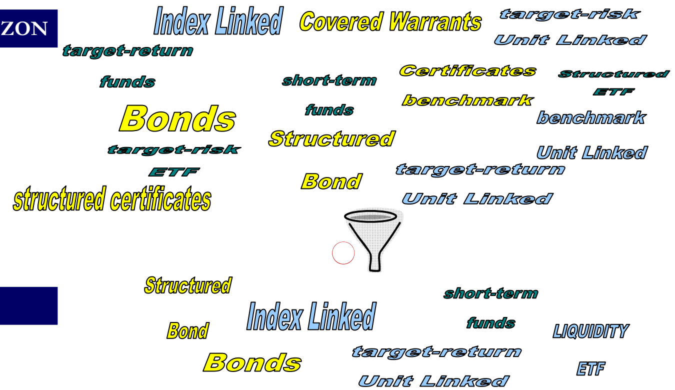
RETURNS

INVESTMENT HORIZON
(less than 3 years)



INVESTMENT HORIZON
(less than 3 years)

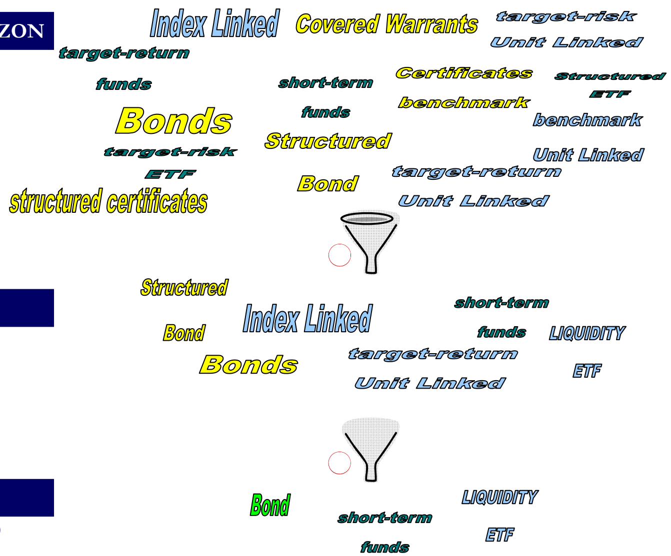
RISKS
(medium-low)



INVESTMENT HORIZON
(less than 3 years)

RISKS
(medium-low)

RETURNS
(maximum return)



RETURNS | **RISKS** | **INVESTMENT HORIZON**

… allow the investor to match his needs with the features of the financial products and to make an informed investment decision

RETURNS | **RISKS** | **INVESTMENT HORIZON**

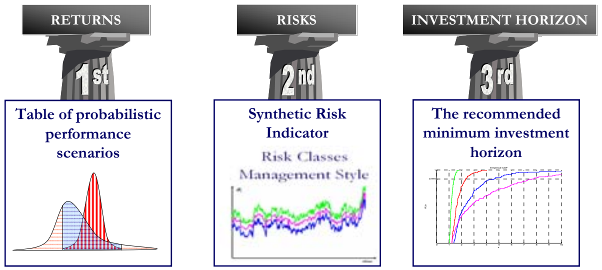
… allow the investor to match his needs with the features of the financial products and to make an informed investment decision

PREVENT MISBUYING

RETURNS | **RISKS** | **INVESTMENT HORIZON**

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

Identification and representation of risk-reward by a three-pillars approach



The three-pillars approach must be implemented via the proprietary models of risk management used by the industry, according to the general principles specified in the transparency regulation.

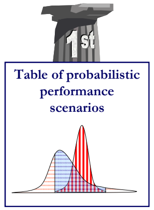
OUTLINE

Non-equity investment products

- First Pillar
- Second Pillar
- Third Pillar

Financing products

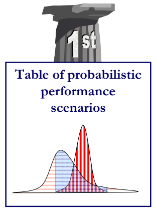
Identification and representation of risk-reward by a three-pillars approach



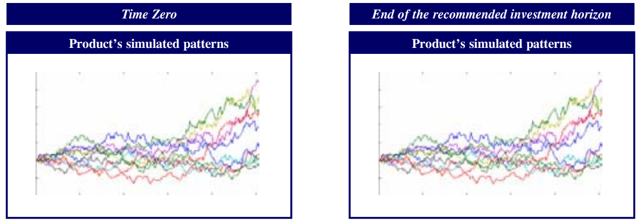
Performance Risk
w.r.t. the risk-free asset
under
the risk-neutral probability measure

… illustrates the unbundling of the price of the financial products at the time of subscription and provides clear and concise information about the possible outcomes and costs.

Identification and representation of risk-reward by a three-pillars approach



Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon



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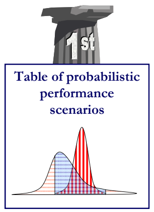

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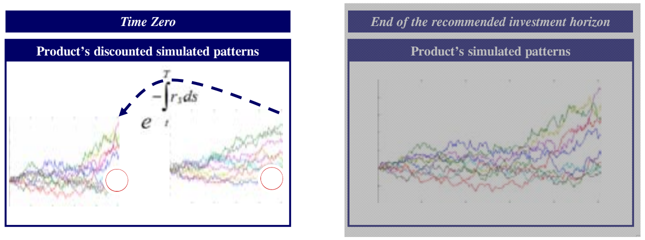

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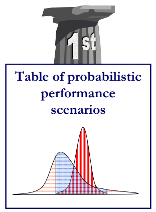

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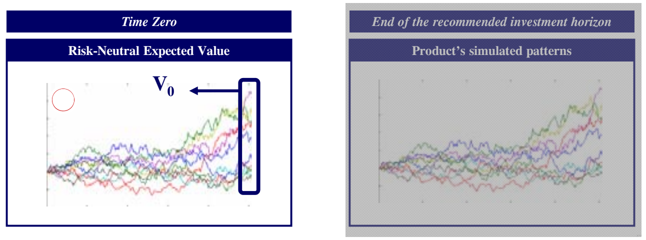

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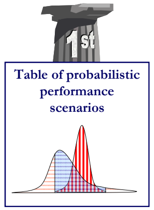


Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon

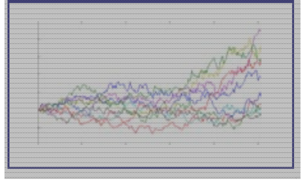
Time Zero

Financial investment table

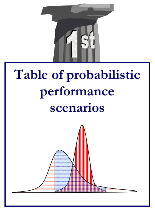
(A) Invested Capital
(B) Costs
(C) = (A) + (B) Notional Capital

End of the recommended investment horizon

Product's simulated patterns



Identification and representation of risk-reward by a three-pillars approach



Unbundling of the financial investment at time zero

Financial investment table

(A) Invested Capital
(B) Costs
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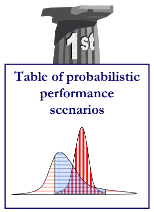
Identification and representation of risk-reward by a three-pillars approach



Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon

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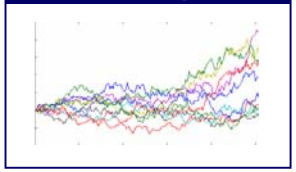
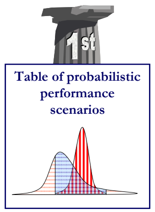


Identification and representation of risk-reward by a three-pillars approach



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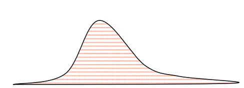
Returns probability distribution



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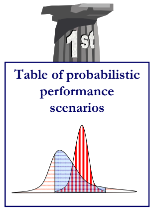

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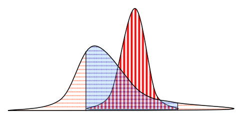
Probabilistic comparison wrt the risk-free asset



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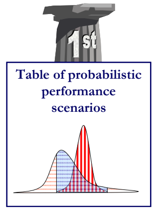

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Time Zero

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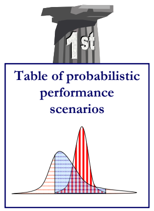
End of the recommended investment horizon

Table of probabilistic performance scenarios

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	%	€
The performance is positive but lower than risk-free asset	%	€
The performance is positive and in line with risk-free asset	%	€
The performance is positive and higher than risk-free asset	%	€

Identification and representation of risk-reward by a three-pillars approach



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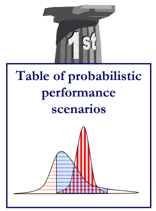

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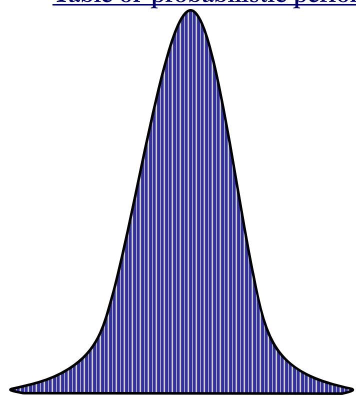
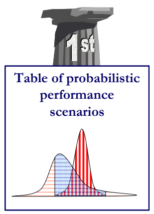


Probability Distribution of the final value of the Notional Capital invested in the risk-free asset

Identification and representation of risk-reward by a three-pillars approach



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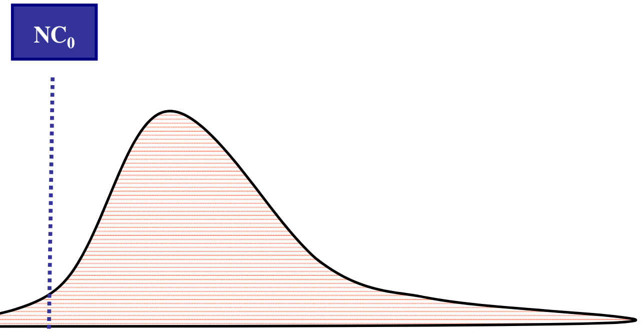
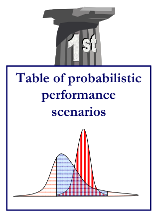


Probability Distribution of the final value of the Invested Capital

Identification and representation of risk-reward by a three-pillars approach



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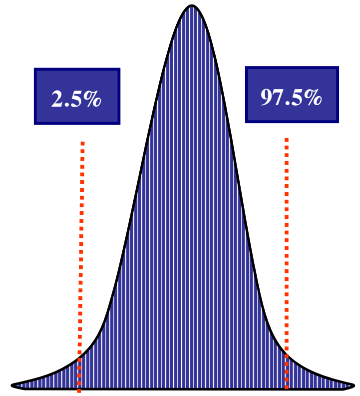

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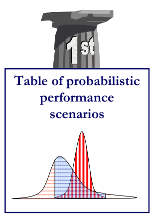

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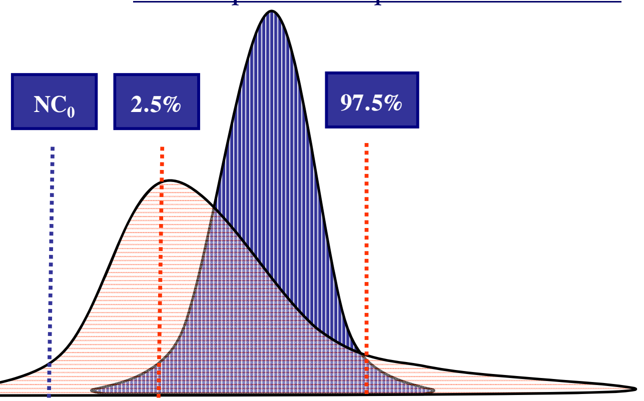

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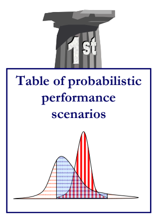

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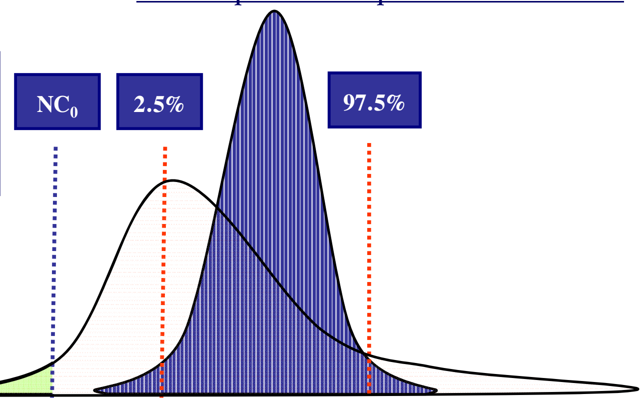
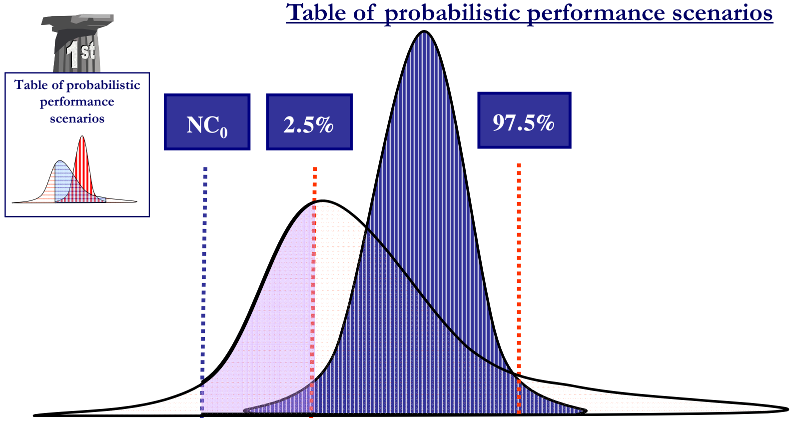


Identification and representation of risk-reward by a three-pillars approach

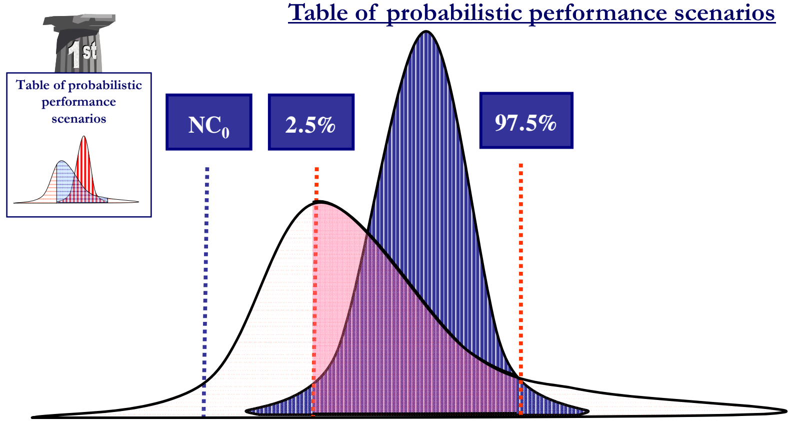
Table of probabilistic performance scenarios



The performance is positive but lower than the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

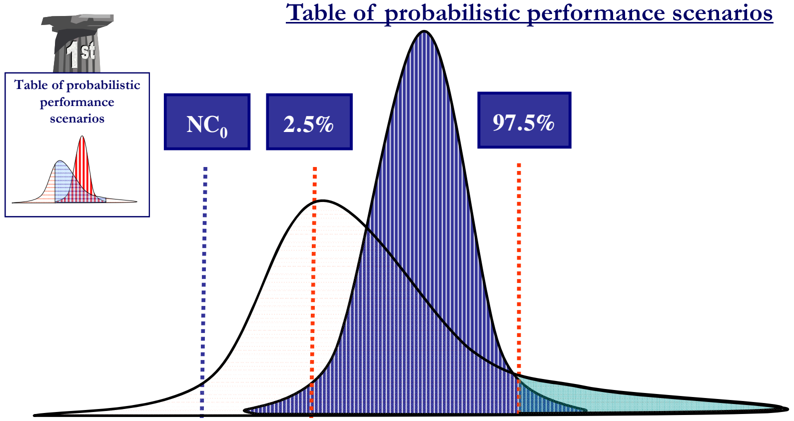
Table of probabilistic performance scenarios



The performance is positive and in line with the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

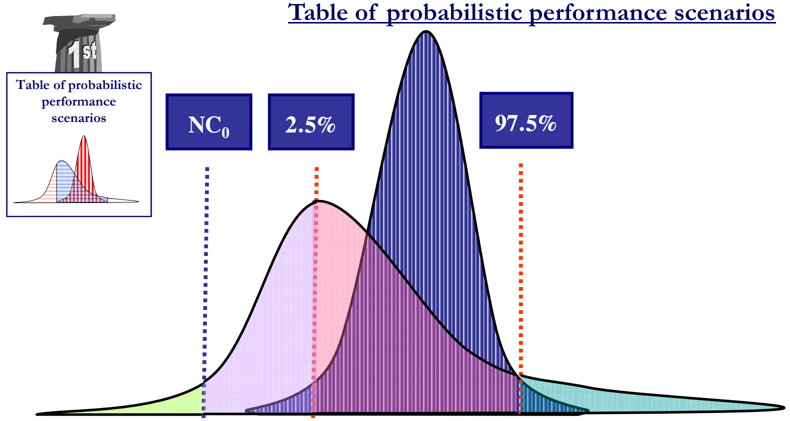
Table of probabilistic performance scenarios



The performance is positive higher than the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

Table of probabilistic performance scenarios



The performance is negative

The performance is positive but lower than the risk-free asset

The performance is positive and in line with the risk-free asset

The performance is positive higher than the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

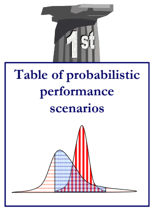


Table of probabilistic performance scenarios

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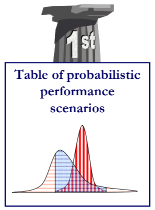

Table of probabilistic performance scenarios
Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon

Time Zero

Financial investment table

(A) Invested Capital
(B) Costs
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End of the recommended investment horizon

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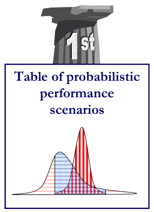
1:1 Relationship

Identification and representation of risk-reward by a three-pillars approach



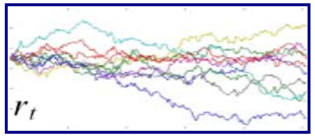
Model Risk Assessment

The Recommended Time Horizon has a significant influence on the choice of the model

For Time Horizons greater than 1 year…..

I

II



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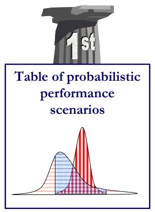


Model Risk Assessment

The Recommended Time Horizon has a significant influence on the choice of the model

Two possible choices…

I

HESTON

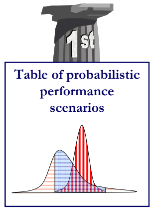
Stochastic Volatility Model

II

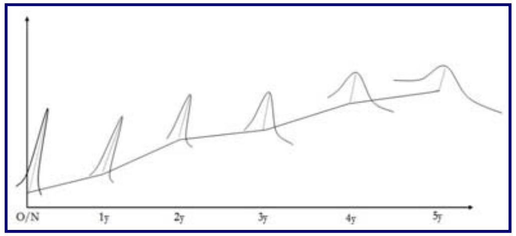
BAKSHI CAO CHEN

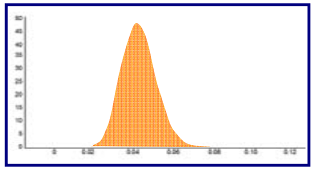
Stochastic Volatility - Stochastic Interest Rate - Jump Model

Identification and representation of risk-reward by a three-pillars approach

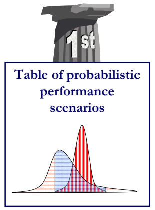


Step 1: Calculation of the Probability Distribution of the Notional Capital at the end of recommended time horizon



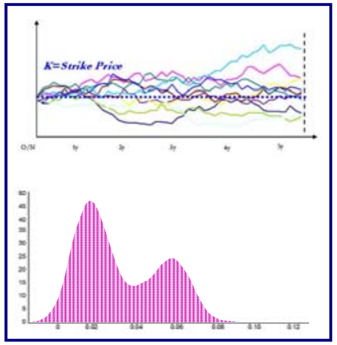


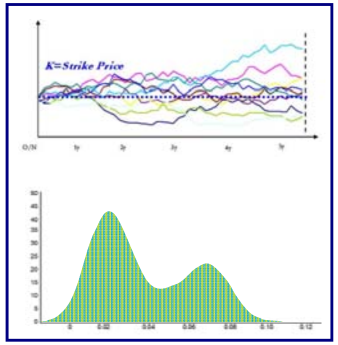
Identification and representation of risk-reward by a three-pillars approach



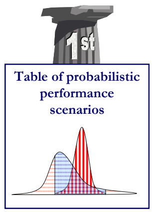
Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Heston **BCC**



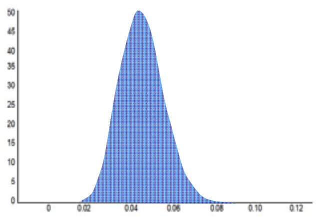


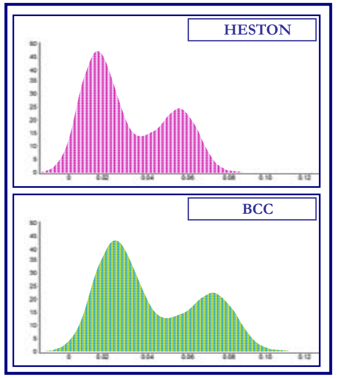
Identification and representation of risk-reward by a three-pillars approach



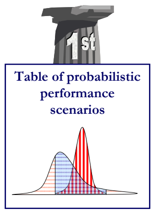
Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Probability Distribution of the Risk-Free Asset





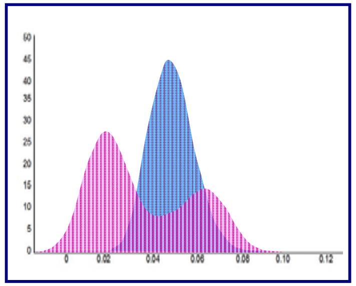
Identification and representation of risk-reward by a three-pillars approach

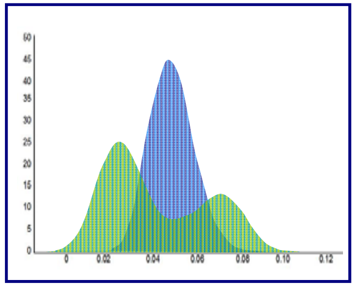


Step 3: Probabilistic comparison with the Risk-Free Asset

Analysing the two probability distributions…

Heston **BCC**

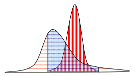




Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



Step 3: Probabilistic comparison with the Risk-Free Asset

The following output is obtained:

Heston

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	4.80 %	97.44 €
The performance is positive but lower than risk-free asset	52.1 %	100.01 €
The performance is positive and in line with risk-free asset	18.42 %	115.63 €
The performance is positive and higher than risk-free asset	24.68 %	135.07 €

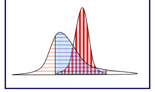
BCC

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	7.20 %	96.92 €
The performance is positive but lower than risk-free asset	48.81 %	100.11 €
The performance is positive and in line with risk-free asset	16.2 %	117.22 €
The performance is positive and higher than risk-free asset	27.79 %	137.56 €

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



Assessing the model risk

Heston

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$|\Delta| = 2{,}40\%$

Identification and representation of risk-reward by a three-pillars approach

1st

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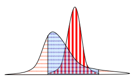

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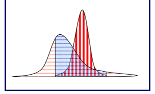
$|\Delta| = 3{,}29\%$

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



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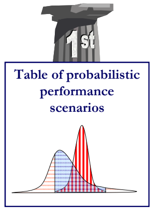
$|\Delta| = 2{,}22\%$

Identification and representation of risk-reward by a three-pillars approach



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$|\Delta| = 3{,}11\%$

OUTLINE

Non-equity investment products

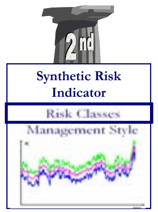
First Pillar

Second Pillar

Third Pillar

Financing products

Identification and representation of risk-reward by a three-pillars approach

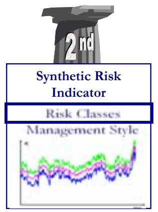


Synthetic Risk Indicator

… provides a description, on a qualitative scale, of the risk level of the financial products based on volatility measures.

… represents in an explicit way the riskiness of the product embedded in the probabilistic performance scenarios of the first pillar.

Identification and representation of risk-reward by a three-pillars approach

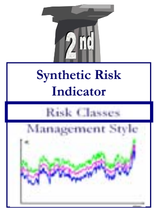


Synthetic Risk Indicator

Six Qualitative Risk Classes

Risk Classes
Low
Medium-Low
Medium
Medium-High
High
Very High

Identification and representation of risk-reward by a three-pillars approach



Synthetic Risk Indicator

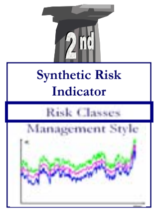
The model has to take in account the following steps …

Time evolution of the Volatility

↓

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Identification and representation of risk-reward by a three-pillars approach



Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Calibration of Volatility Intervals

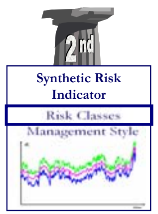
↓

Step 1: Definition of Loss Intervals

Step 2: Mapping of Loss Intervals to the corresponding Volatility Intervals

Step 3: *Fine-tuning* of Volatility Intervals

Identification and representation of risk-reward by a three-pillars approach



Step 1: **Definition of Loss Intervals**

What is a loss in a financial investment?

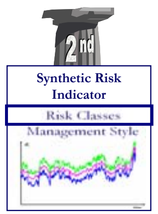
RISK NEUTRALITY PRINCIPLE

↓

$$\text{LOSS} \in (-100\%, \overline{r^{rf}}\,]$$

$\overline{r^{rf}}$ = average of the probability distribution of the risk-free rate

Identification and representation of risk-reward by a three-pillars approach

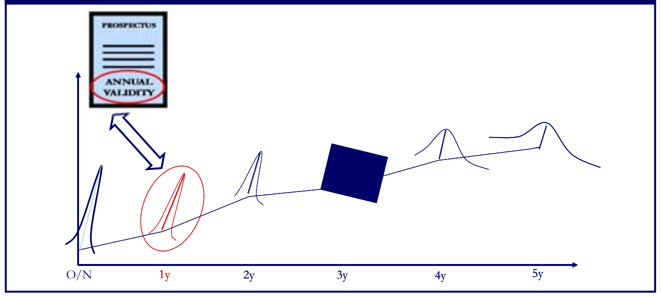


Step 1: **Definition of Loss Intervals**

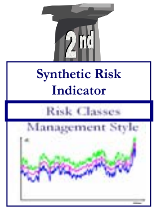
given the risk-free yield curve and the associated volatility surface…

↓

Risk-free yield curve and *volatility surfaces*



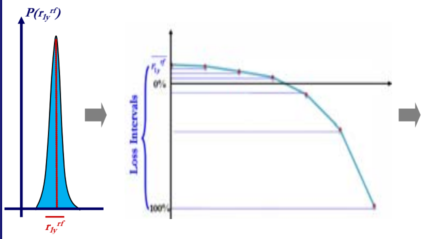
Identification and representation of risk-reward by a three-pillars approach



Step 1: Definition of Loss Intervals

the corresponding annual loss interval (multiple of r_{1y}^{rf} according to an exponential function) is associated to each risk class

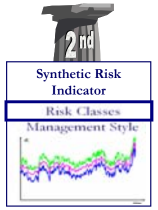
From *risk-free* yield curve to one year loss intervals



Risk Classes	Loss Intervals L_{min}	L_{max}
low	$_0L_{1,min}$	$_0L_{1,max}$
medium-low	$_0L_{2,min}$	$_0L_{2,max}$
medium	$_0L_{3,min}$	$_0L_{3,max}$
medium-high	$_0L_{4,min}$	$_0L_{4,max}$
high	$_0L_{5,min}$	$_0L_{5,max}$
very high	$_0L_{6,min}$	$_0L_{6,max}$

FRONTIERS IN FINANCIAL MARKETS MATHEMATICS
Bologna, 13-17 LUGLIO 2009
Quantitative measures for an unified approach to risks disclosure in financial products

Identification and representation of risk-reward by a three-pillars approach



Step 2: Mapping into Initial Volatility Intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	$_0L_{1,min}$	$_0L_{1,max}$
medium-low	$_0L_{2,min}$	$_0L_{2,max}$
medium	$_0L_{3,min}$	$_0L_{3,max}$
medium-high	$_0L_{4,min}$	$_0L_{4,max}$
high	$_0L_{5,min}$	$_0L_{5,max}$
very high	$_0L_{6,min}$	$_0L_{6,max}$

Risk Classes	Volatility Intervals σ_{min}	σ_{max}
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medium -high	$_0\sigma_{4,min}$	$_0\sigma_{4,max}$
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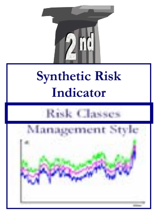

Step 3: *Fine-tuning* of Volatility Intervals

TOOLS

- √ **GARCH Diffusive Models**
- √ **Non linear Stochastic Programming**

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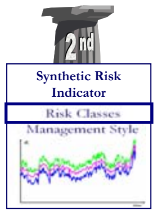
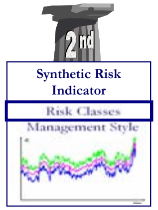


Step 3: *Fine-tuning* of Volatility Intervals: GARCH Diffusive Models

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

The Weak Convergence Theorem on R^2

The sequence $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$, converges weakly for h 0 to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

Condition

n. 1

If $\exists$ a $\delta>0$ s.t.:

$$\lim_{h\downarrow 0}\begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$$

then $\exists$

$a(x,t)$

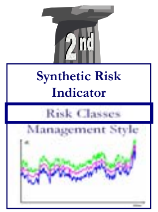
$$\lim_{h\downarrow 0}\begin{pmatrix} a_h(x_1,t) & a_h((x_1,x_2),t) \\ a_h((x_2,x_1),t) & a_h(x_2,t) \end{pmatrix} = \begin{pmatrix} a(x_1,t) & 0 \\ 0 & a(x_2,t) \end{pmatrix}$$

s.t.

$b(x,t)$

$$\lim_{h\downarrow 0}\begin{pmatrix} b_h(x_1,t) \\ b_h(x_2,t) \end{pmatrix} = \begin{pmatrix} b(x_1,t) \\ b(x_2,t) \end{pmatrix}$$

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

Condition

n. 2

$\exists\ \sigma(x,t)$ s.t.: $\forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1,$

it holds

$$\begin{pmatrix} \sigma(x_1,t) & 0 \\ 0 & \sigma(x_2,t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1,t)} & 0 \\ 0 & \sqrt{a(x_2,t)} \end{pmatrix}$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

Conditions

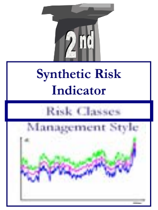
n. 3

For h$\downarrow$0, X_0^h converges in distribution to a random variable X_0 with probability measure v_0 on $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$

n. 4

v_0, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution v_0 , a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

The Continuous Limit of the M-GARCH(1,1)

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \widetilde{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

$\widetilde{Z}_k$ and Z_k are i.i.d $N(0,1)$

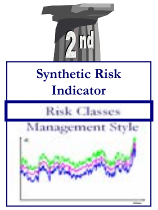
Weak Convergence Theorem

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right) dt + 2 |\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

Identification and representation of risk-reward by a three-pillars approach



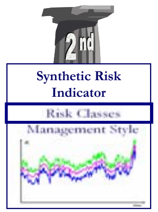
Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

The Prediction Interval for the Volatility

key point

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a *Predictive Interval for σ_t*

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

The Prediction Interval for the Volatility

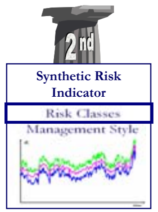
distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d \ln \sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right] dt + 2 |\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

O-U process

$$\ln \sigma_t^2 \sim N\left[\left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln|Z_t|)}{(\beta_1 - 1)}\right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln|Z_t|)}{(\beta_1 - 1)} ; \sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1 - 1)} \left(e^{2(\beta_1 - 1)} - 1\right)}\right]$$

Identification and representation of risk-reward by a three-pillars approach

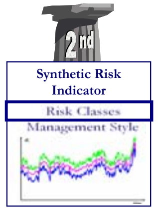


Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

matching of the first two conditional moments

$$\begin{aligned} \ln \sigma_k^2 - \ln \sigma_{k-1}^2 = & \frac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1 - 1)} - 1\right)}{\beta_1 - 1} - \\ & - 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) + \\ & + \left(e^{(\beta_1 - 1)} - 1\right) \ln \sigma_{k-1}^2 + \\ & + 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln |Z_{k-1}| \end{aligned}$$

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

maximum likelihood estimation

setting:

$$Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$$

$$a = \frac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} - E(\ln|Z_{k-1}|)\,|\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)}-1\right)}{(\beta_1-1)}}$$

$$b = \left(e^{(\beta_1-1)}-1\right)$$

$$c = |\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)}-1\right)}{(\beta_1-1)}}$$

$$X_{k-1} = \ln \sigma_{k-1}^2$$

$$Z = \ln|Z_{k-1}|$$

FRONTIERS IN FINANCIAL MARKETS MATHEMATICS
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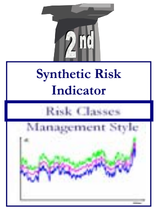
Quantitative measures for an unified approach to risks disclosure in financial products

Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

maximum likelihood estimation

leads to:

$$Y_k = a + bX_{k-1} + cZ$$

log-likelihood function

$$\ln L(Y;\beta_0,\beta_1) = n\ln\left(\frac{2}{c\sqrt{2\pi}}\right) + \sum_{k=1}^{n}\left(\frac{Y_k - a - bX_{k-1}}{c} - \frac{1}{2}e^{2\left(\frac{Y_k - a - bX_{k-1}}{c}\right)}\right)$$

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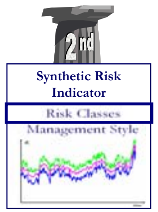

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

$$P\left(\begin{array}{c} -z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)} \\ \le \ln\sigma_t^2 \le \\ z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)} \end{array}\right) = \alpha$$

hence:

$$\left[\sigma_{t,\min}^G;\sigma_{t,\max}^G\right] = \left[\begin{array}{l} e^{\frac{-z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0-1.27048\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0-1.27048\beta_1}{(\beta_1-1)}}{2}}; \\ e^{\frac{z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0-1.27048\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0-1.27048\beta_1}{(\beta_1-1)}}{2}} \end{array}\right]$$

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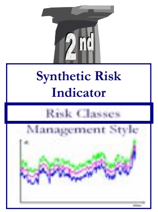
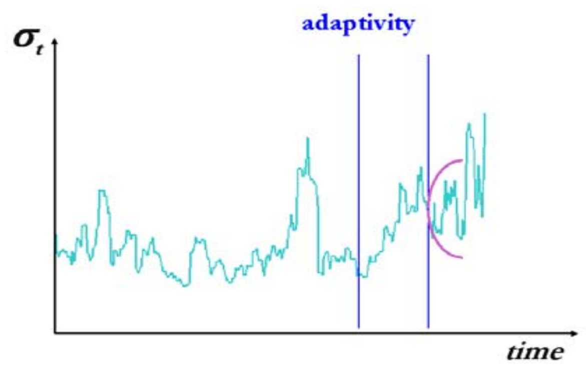
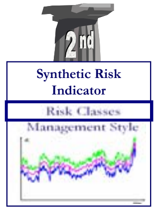
Identification and representation of risk-reward by a three-pillars approach



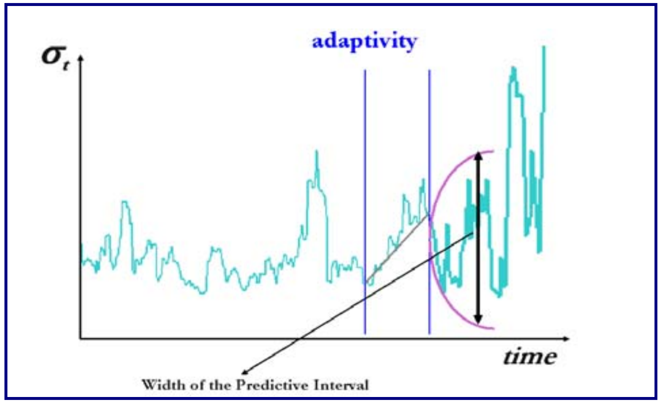
Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**



Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

√ **Non linear Stochastic Programming**

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

1. The Product Pattern is simulated for each Initial Volatility Interval

Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	${}_0\sigma_{1,min}$	${}_0\sigma_{1,max}$
medium-low	${}_0\sigma_{2,min}$	${}_0\sigma_{2,max}$
medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium -high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,min}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

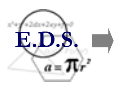
Identification and representation of risk-reward by a three-pillars approach

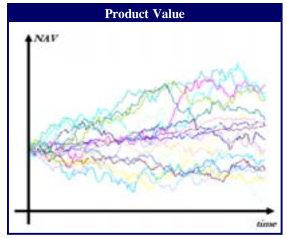
Step 3: *Fine-tuning* of Volatility Intervals

1. The Product Pattern is simulated for each Initial Volatility Interval

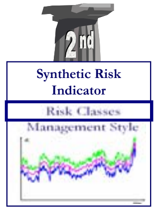
RISK NEUTRALITY

Initial Volatility Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$





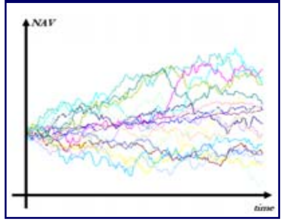
Identification and representation of risk-reward by a three-pillars approach



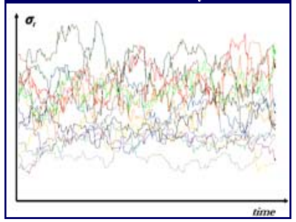
Step 3: *Fine-tuning* of Volatility Intervals

2. Determination of the Time Series of the Annualized Volatility of Product Daily Returns

Product Value



Annualized Volatiliy



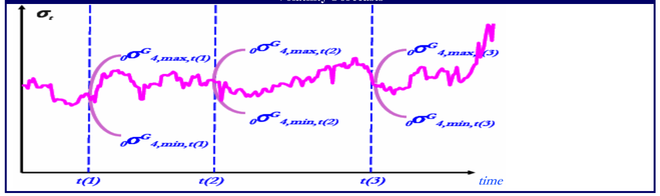
Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

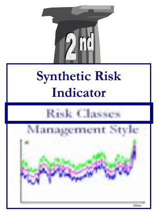
3. For each trajectory the Volatility forecast band is calculated using GARCH Diffusive Models

$$\left[\sigma^G_{t,\min};\sigma^G_{t,\max}\right] = e^{\frac{-z_{\alpha/2}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma^2_{t-1}+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}};\; e^{\frac{z_{\alpha/2}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma^2_{t-1}+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}}$$

Volatility Forecasts



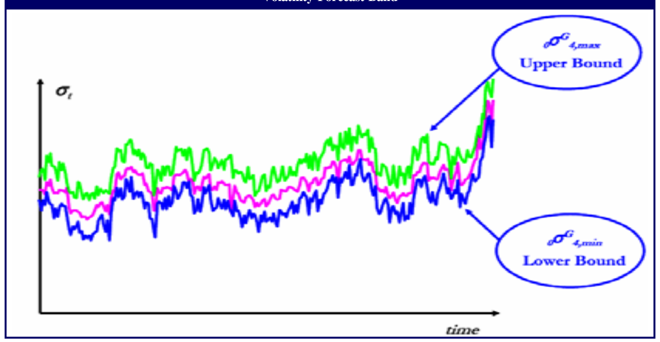
Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals

3. For each trajectory the Volatility forecast band is calculated using GARCH Diffusive Models

Volatility Forecast Band

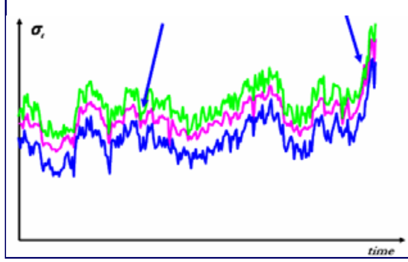


Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

4. Validation of Initial Volatility Interval through an iterative procedure that minimizes the number of observations outside the band

Volatility Forecast Band



$$\Delta = \frac{[\text{Tot.} > {}_0\sigma^G_{4,max}] + [\text{Tot.} < {}_0\sigma^G_{4,min}]}{n*250}$$

$$\Delta_{up} = \frac{[\text{Tot.} > {}_0\sigma^G_{4,max}]}{n*250}$$

$$\Delta_{down} = \frac{[\text{Tot.} < {}_0\sigma^G_{4,min}]}{n*250}$$

$\Delta > 5\%$

The Interval is updated

Identification and representation of risk-reward by a three-pillars approach

Synthetic Risk Indicator
Risk Classes
Management Style

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

BEGIN PROCEDURE

Identification and representation of risk-reward by a three-pillars approach

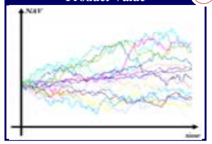
Synthetic Risk Indicator
Risk Classes
Management Style

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Identification and representation of risk-reward by a three-pillars approach

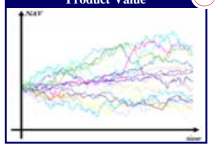
Synthetic Risk Indicator
Risk Classes
Management Style

Step 3: *Fine-tuning* of Volatility Intervals

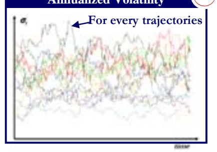
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility



Identification and representation of risk-reward by a three-pillars approach

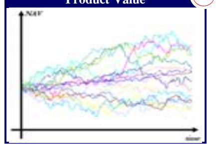
Synthetic Risk Indicator
Risk Classes
Management Style

Step 3: *Fine-tuning* of Volatility Intervals

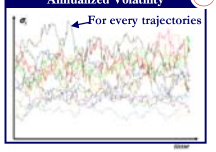
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

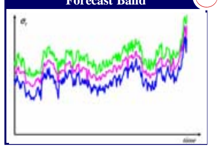
Product Value



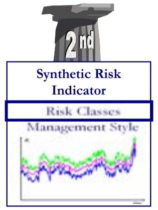
Annualized Volatility



Forecast Band

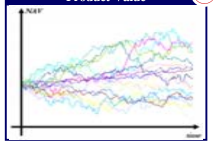


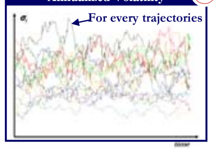
Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

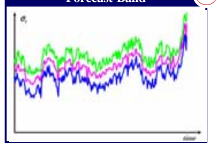
Product Value


Annualized Volatility


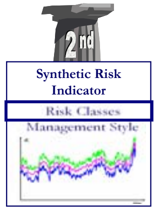
Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

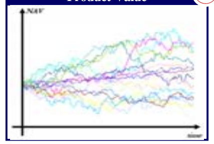
Forecast Band


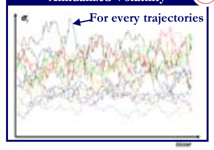
Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value


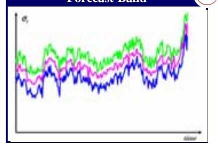
Annualized Volatility


$\Delta > 5\%$
e
$\Delta_{up} > \Delta_{down}$

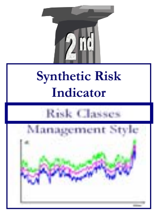
Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band


Identification and representation of risk-reward by a three-pillars approach

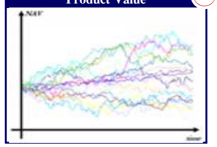


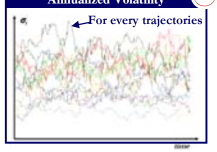
Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Update
$[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$

$\Delta > 5\%$
e
$\Delta_{up} > \Delta_{down}$

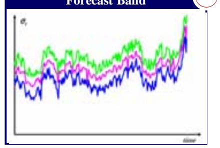
Product Value


Annualized Volatility


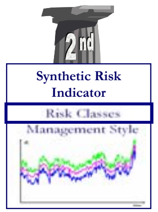
Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

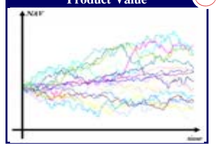
Forecast Band


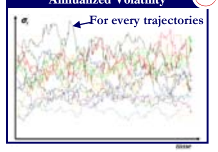
Identification and representation of risk-reward by a three-pillars approach



Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value


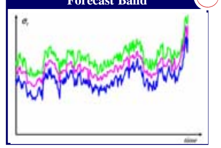
Annualized Volatility


$\Delta > 5\%$
e
$\Delta_{up} < \Delta_{down}$

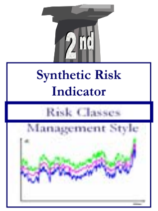
Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band


Identification and representation of risk-reward by a three-pillars approach



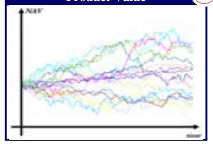
Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

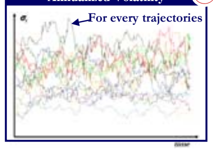
Update
$[{}_1\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

$\Delta > 5\%$ e $\Delta_{up} < \Delta_{down}$

Product Value



Annualized Volatility

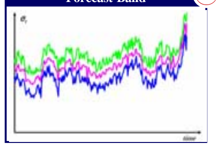


Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

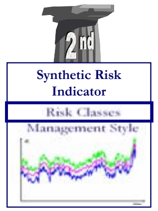
VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band

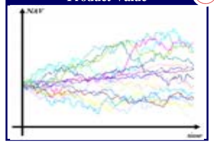


Identification and representation of risk-reward by a three-pillars approach

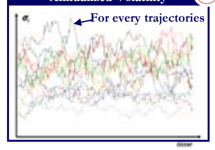


Step 3: *Fine-tuning* of Volatility Intervals

Product Value



Annualized Volatility



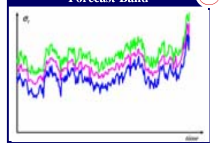
$\Delta \leq 5\%$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band



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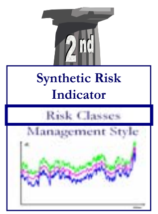
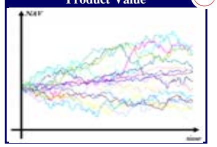


Step 3: *Fine-tuning* of Volatility Intervals

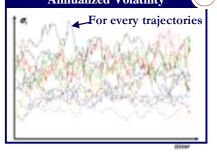
END PROCEDURE
$[{}_k\sigma_{4,min} \quad {}_k\sigma_{4,max}]$
$=$
$[\sigma_{4,min} \quad \sigma_{4,max}]$

$\Delta \leq 5\%$

Product Value



Annualized Volatility



Initial Interval
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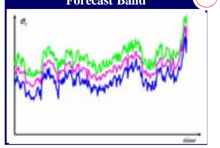

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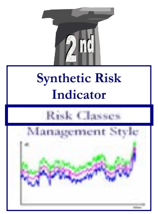
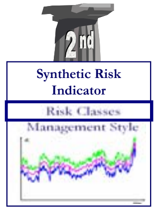


Step 3: *Fine-tuning* of Volatility Intervals

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0,01%	0,49%
Medium-Low	0,50%	1,59%
Medium	1,60%	3,99%
Medium-High	4,00%	9,99%
High	10,00%	24,99%
Very High	25,00%	>25,00%

Identification and representation of risk-reward by a three-pillars approach



Synthetic Risk Indicator

Time evolution of the Volatility

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0,01%	0,49%
Medium-Low	0,50%	1,59%
Medium	1,60%	3,99%
Medium-High	4,00%	9,99%
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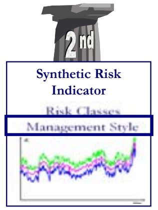
Identification and representation of risk-reward by a three-pillars approach



Synthetic Risk Indicator

The model has to take in account the following steps …

Benchmark Products

Time evolution of the "intensity" of the Management Style

Mapping of each Volatility Interval into corresponding Intervals of a suitable Volatility Measure for every Management Style

Identification and representation of risk-reward by a three-pillars approach

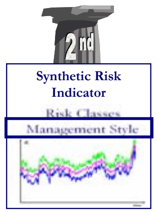
Benchmark Products
"Intensity" and "Direction" of the Management Style

Three different classes of the active management

Active Management Style Classes

- Limited
- Intermediate
- Considerable

Identification and representation of risk-reward by a three-pillars approach



Synthetic Risk Indicator

The model has to take in account the following steps …

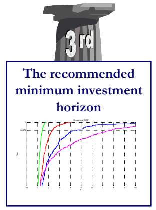
Benchmark Products

Time evolution of the "intensity" of the Management Style

Risk Classes	Delta-Vol Intervals					
	Limited		Intermediate		Considerable	
	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$
Low	-0.118%	0.118%	-0.176%	0.176%	-0.235%	0.235%
Medium-Low	-0.239%	0.239%	-0.358%	0.358%	-0.477%	0.477%
Medium	-0.600%	0.600%	-0.900%	0.900%	-1.200%	1.200%
Medium-High	-1.250%	1.250%	-1.875%	1.875%	-2.500%	2.500%
High	-3.125%	3.125%	-4.668%	4.668%	-6.249%	6.249%
Very High	-6.250%	6.250%	-9.375%	9.375%	-12.500%	12.500%

OUTLINE

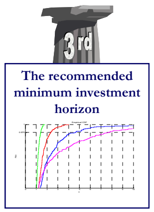
Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

Investment period which can be deemed appropriate having regard to the risk-reward profile and to the costs of the product

Identification and representation of risk-reward by a three-pillars approach

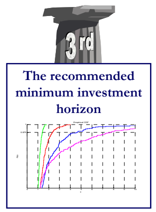


The recommended minimum investment horizon

The recommended minimum investment horizon crucially depends on types of financial products…

Risk Target Products | **Return Target Products** | **Benchmark Products**

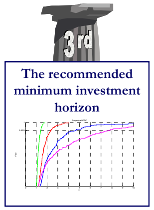
Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

… for return target products and for guaranteed products the recommended minimum investment horizon is inherent to their financial engineering, as:

Identification and representation of risk-reward by a three-pillars approach

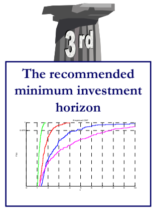


The recommended minimum investment horizon

… for performance target products and for guaranteed products the recommended minimum investment horizon is inherent to their financial engineering, as:

> the recommended minimum investment horizon is
>
> =
>
> the period of validity (or the time to maturity) of their target/guarantee mechanisms

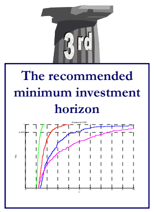
Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

… for risk target products or benchmark products is calculated as the break-even time of the financial investment, i.e. the time needed to recover the initial charges and to offset the ongoing costs at least once, from a probabilistic perspective.

Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

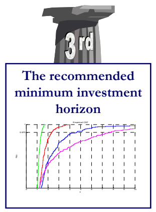
Formally speaking, the probability of the event

> **The investiment recovers the initial charges and offsest the ongoing costs at least once**

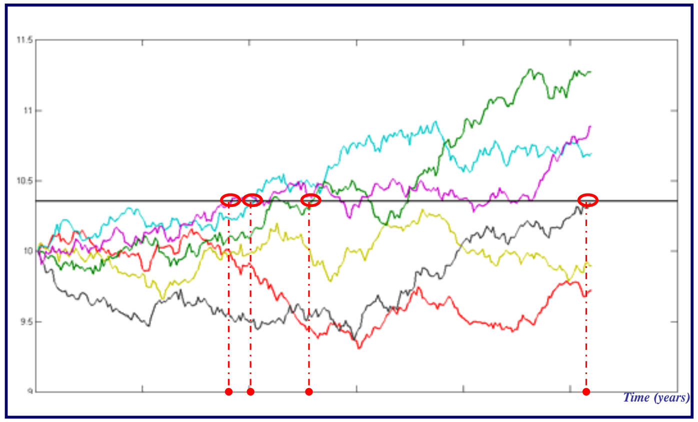
can be calculated using the mathematical concept of

First Hitting Time

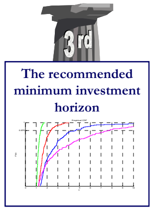
Identification and representation of risk-reward by a three-pillars approach



First Hitting Time of a Structured Product:
first time (expressed in years) at which the value of the product recovers the initial cost and offsets the ongoing costs



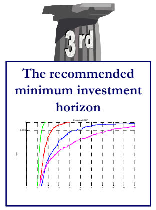
Identification and representation of risk-reward by a three-pillars approach



$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \leq T \right] = \alpha \right\}$$

Computational Steps

Identification and representation of risk-reward by a three-pillars approach



The probability of the event

The investiment recovers the initial charges and offsest the ongoing costs at least once

is perfectly represented using the cumulative distribution of first hitting times, i.e:

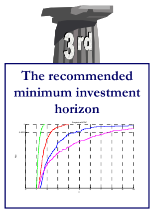
$$\mathrm{P}\left[t^* \leq T \right] = X\%$$

where

$$t^* = \inf\left[t \in \mathbb{R} : CI_t > CN \right]$$

is the first hitting time

Identification and representation of risk-reward by a three-pillars approach



The probability of the event

The investiment recovers the initial charges and offsest the ongoing costs at least once

given a level of confidence α, identifies univocally a time T on the cumulative distribution of first hitting times, i.e.:

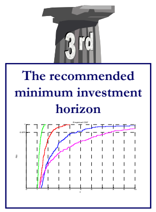
$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \leq T \right] = \alpha \right\}$$

where

$$t^* = \inf\left[t \in \mathbb{R} : CI_t > CN \right]$$

is the first hitting time

Identification and representation of risk-reward by a three-pillars approach



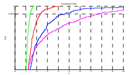
$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \leq T \right] = \alpha \right\}$$

is defined as the recommended minimum investiment horizon

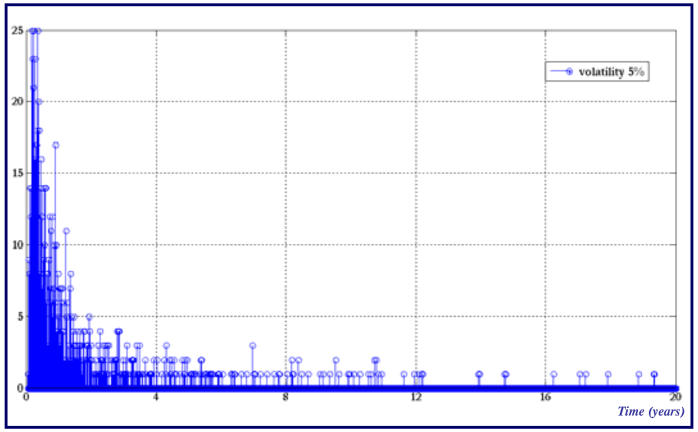
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



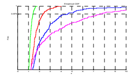
1. The First Hitting Time Distribution of the Structured Product is calculated:



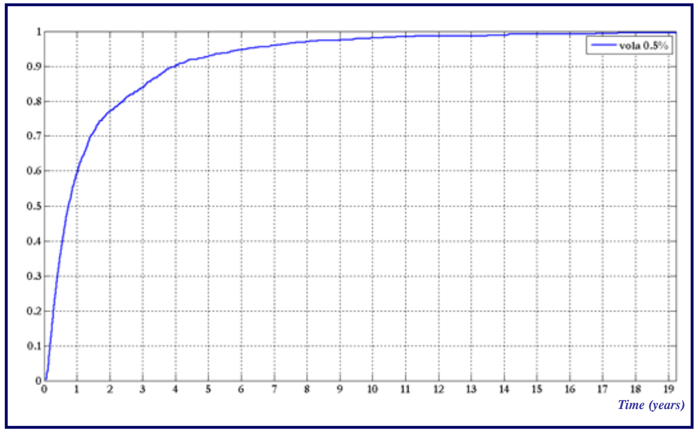
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



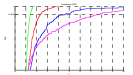
2. The First Hitting Time Cumulative Distribution of the Structured Product is then represented:



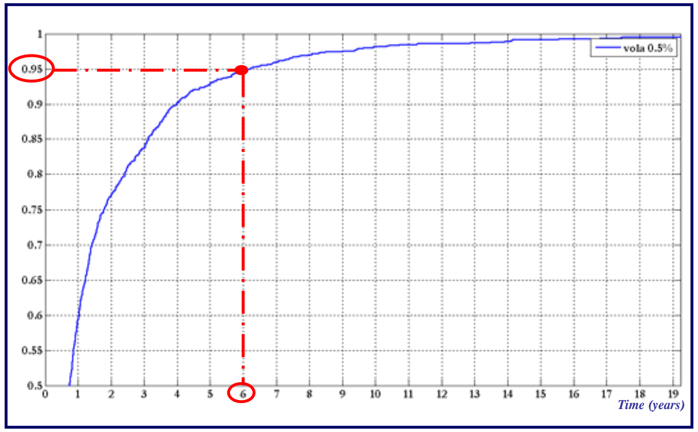
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



3. The level of confidence α identifies univocally T on the cumulative distribution of first hitting times:



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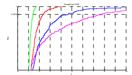
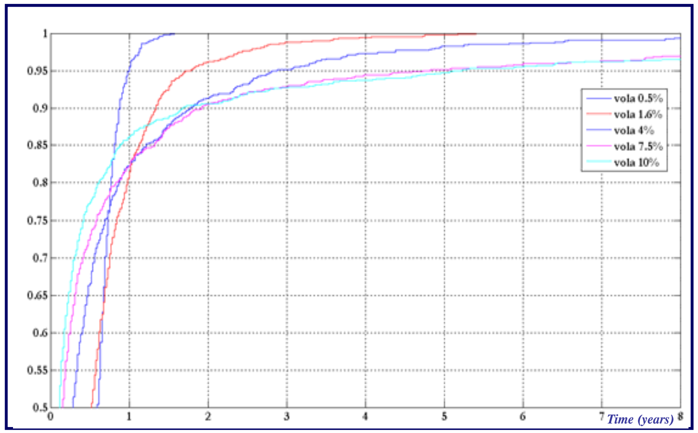


The recommended minimum investiment horizon is heavily depedent from the measured level of volatility:

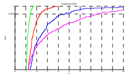


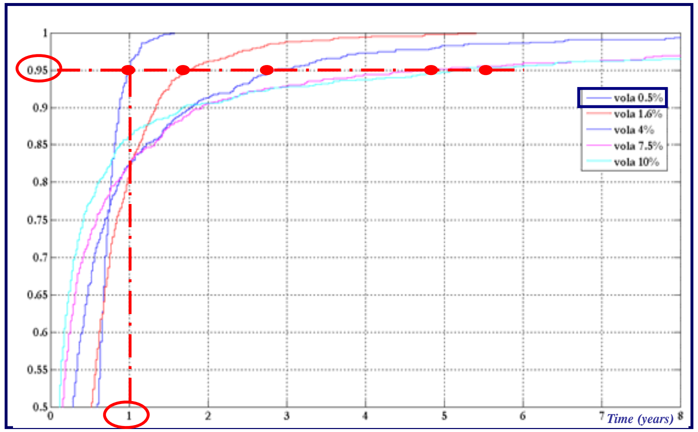
Identification and representation of risk-reward by a three-pillars approach

The recommended minimum investiment horizon is heavily depedent from the measured level of volatility:

3rd

The recommended minimum investment horizon





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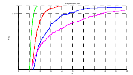

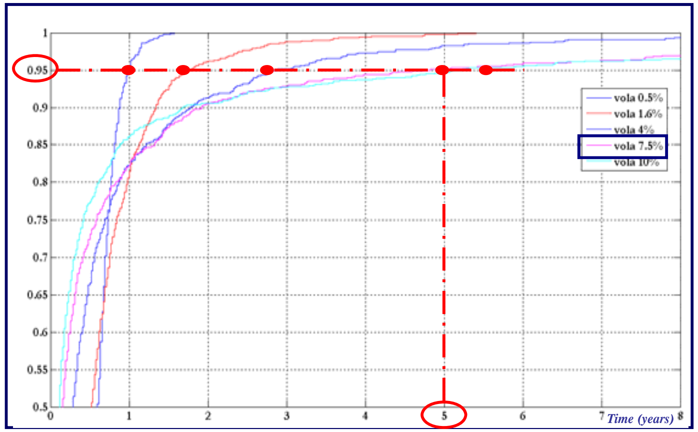

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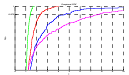

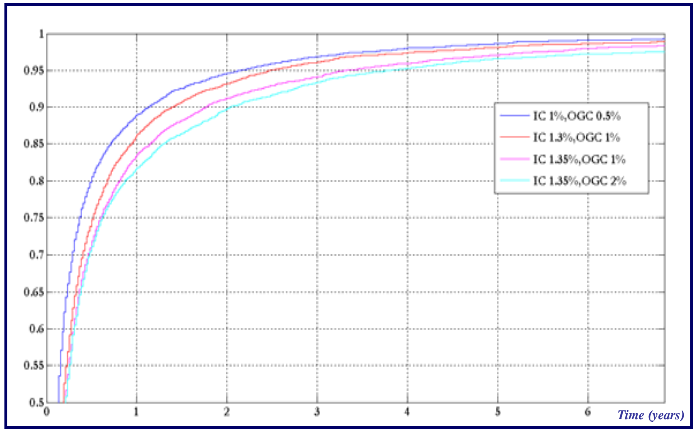

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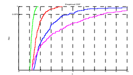

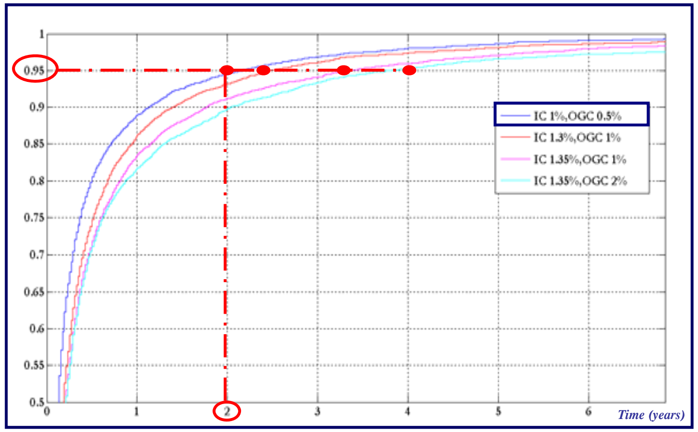

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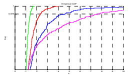

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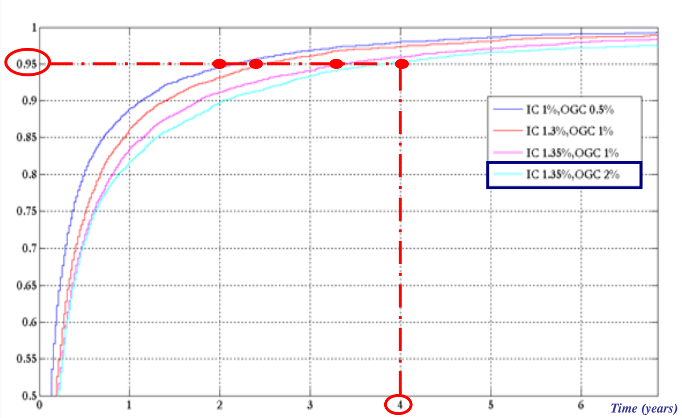

PROBABILITY, VOLATILITY AND COSTS

An *analitical tool* in order to better appreciate the relationship between probability, volatility and costs

PROBABILITY, VOLATILITY AND COSTS

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In a Black-Scholes-Merton world under the hypothesis…..

Constant interest rate

Constant Volatility

PROBABILITY, VOLATILITY AND COSTS

An *analitical tool* in order to better appreciate the relationship between probability, volatility and costs

In a Black-Scholes-Merton world under the hypothesis…..

Constant interest rate

Constant Volatility

First hitting time cumulative probability can be determined through a closed formula (Karatzas-Shreve – 1991):

PROBABILITY, VOLATILITY AND COSTS

An *analitical tool* in order to better appreciate the relationship between probability, volatility and costs

In a Black-Scholes-Merton world under the hypothesis…..

Constant interest rate

Constant volatility

First hitting time cumulative probability can be determined through a closed formula (Karatzas-Shreve – 1991):

PROBABILITY, VOLATILITY AND COSTS

First hitting time cumulative probability can be determined through a closed formula (Karatzas-Shreve – 1991):

$$T = \left\{ t \in \mathbb{R}^+ : \mathrm{P}\left[t^* \le T \right] = \alpha \right\}$$

$$\mathrm{P}\left[t^* \le T \right] = N\left(d_2\left(\frac{CI_0}{CN_0} \right) \right) + \left(\frac{CN_0}{CI_0} \right)^{\frac{2r}{\sigma^2}-1} \cdot N\left(-d_2\left(\frac{CN_0}{CI_0} \right) \right)$$

$$d_2(x) = \frac{\log x + \left(r - cr - \frac{1}{2}\sigma^2 \right) T}{\sigma\sqrt{T}}$$

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

PROBABILITY, VOLATILITY AND COSTS

Asymptotic behavior:

cr: periodic cost

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T \right] = \begin{cases} 1 & se \quad (r - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN_0}{CI_0} \right)^{\frac{2(r-cr)}{\sigma^2}-1} & se \quad (r - cr) \le \frac{1}{2}\sigma^2 \end{cases}$$

PROBABILITY, VOLATILITY AND COSTS

Asymptotic behavior:

cr: periodic cost

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T \right] = \begin{cases} 1 & se \quad (r - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN_0}{CI_0} \right)^{\frac{2(r-cr)}{\sigma^2}-1} & se \quad (r - cr) \le \frac{1}{2}\sigma^2 \end{cases}$$

For products with high periodic costs

PROBABILITY, VOLATILITY AND COSTS

Asymptotic behavior:

cr: periodic cost

$$\lim_{T\to\infty} P\left[t^* \le T\right] = \begin{cases} 1 & se \quad (r-cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN_0}{CI_0}\right)^{\frac{2(r-cr)}{\sigma^2}-1} & se \quad (r-cr) \le \frac{1}{2}\sigma^2 \end{cases}$$

The impact of upfront costs on invested capital determines the asymptotic convergence to a lower statistical significance level

PROBABILITY, VOLATILITY AND COSTS

Asymptotic behavior:

cr: periodic cost

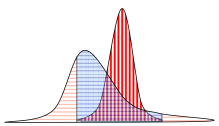
$$\lim_{T\to\infty} P\left[t^* \le T\right] = \begin{cases} 1 & se \quad (r-cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN_0}{CI_0}\right)^{\frac{2(r-cr)}{\sigma^2}-1} & se \quad (r-cr) \le \frac{1}{2}\sigma^2 \end{cases}$$

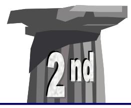
The impact of a volatility increase determines the asymptotic convergence to a lower statistical significance level

RETURNS | **RISKS** | **INVESTMENT HORIZON**

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

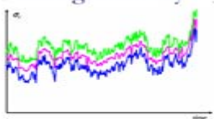
Table of probabilistic performance scenarios

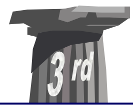




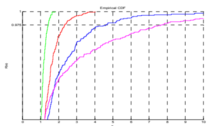
Synthetic Risk Indicator

Risk Classes
Management Style





The recommended minimum investment horizon



OUTLINE

Non-equity investment products

Financing products

Similar approach can be applied also for a comprehensive and transparent disclosure of the risk-profile and the cost impact of any financing products:

- **also when embeds a derivative-like components such as the case of an existing financial position which has to be hedged by market movements.**

CONSOB

Consider for example the position of a firm which has a financing agreement and would like to hedge its position from the increase of the interest rates.

An example?

More than 600 italian municipalities whose indebtness positions have been "hedged" during last few years by means of one or more OTC derivatives.

Disclosure regulation should allow the firm to understand the risk profile and the embedded cost of the trade.

… a simple methodology can be implemented.

CONSOB

The method has to take in account the following steps …

Methodology **<u>Step 1</u>: Identify basic components**

CONSOB

The method has to take in account the following steps …

Methodology **<u>Step 1</u>: Identify basic components**

A debt instrument

CONSOB

The method has to take in account the following steps …

Methodology

Step 1: Identify basic components

A debt instrument

B other hedging instruments

The method has to take in account the following steps …

Methodology

Step 1: Identify basic components

A debt instrument

B other hedging instruments

C = A + B INITIAL PORTFOLIO

The method has to take in account the following steps …

Methodology

Step 1: Identify basic components

A debt instrument

B other hedging instruments

C = A + B INITIAL PORTFOLIO

D derivative contract

The method has to take in account the following steps …

Methodology

Step 1: Identify basic components

A debt instrument

B other hedging instruments

C = A + B INITIAL PORTFOLIO

D derivative contract

E = C + D STRUCTURED PORTFOLIO

The method has to take in account the following steps …

Methodology

Step 1: Identify basic components

- A — debt instrument
- B — other hedging instruments
- C — INITIAL PORTFOLIO
- **D** = E – C — derivative contract
- E — STRUCTURED PORTFOLIO

The method has to take in account the following steps …

Methodology

Step 2: Analysis of the single component of the Structured Portfolio

Decomposition of the STRUCTURED PORTFOLIO in simple component evidencing the cash flow structure for each of them

The method has to take in account the following steps …

Methodology

Step 3: Comparison between INITIAL PORTFOLIO and STRUCTURED PORTFOLIO results

The probabilistic representation of the synthetic swap (i.e. initial portfolio with respect to the structured portfolio value)

QUALIFY

the contract is appropriate

The method has to take in account the following steps …

Methodology

Step 4: Evaluation of the cost of the derivative contract

A coeherent measures of the cost embedded in the contract

QUALIFY

the cost-effectiveness of the contract

OUTLINE

Non-equity investment products

OTC derivatives

An example

Municipality ⟷ Bank

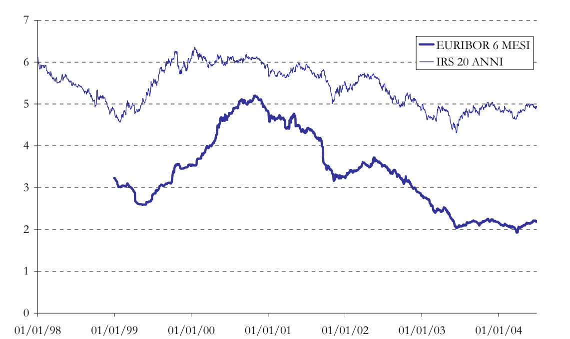
Initial condition

CHARACTERISTICS OF THE FINANCING AGREEMENT

- Trade date: 01/01/1998;
- Termination date: 31/12/2022;
- Notional: € 10.000.000;
- Fixed rate: 6%;
- Amortising: Italian;
- Payment dates: semi-annually.

Economic environment in 2004

INTEREST RATES 1998-2004



Municipality's decision problem

- **since 2001 interest rate are decreasing**
- **financing opportunity cost is increasing**
- **how to save money?**

Municipality ⟷ Bank

Re-negotiation of the financing agreement

PROPOSAL

RENEGOTIATE THE FINANCING AGREEMENT TERMS	TRADE OF A NEW *INTEREST RATE SWAP* CONCTRACT

Structured portfolio components

DEBT INSTRUMENT

- Trade date: 01/01/1998;
- Termination date: 31/12/2022;
- Notional: € 10.000.000;
- Fixed rate: 6% p.a.;
- Amortising: italian;
- Payment dates: semi-annually.

Structured portfolio components

OTHER HEDGING TRANSACTION

NONE

Structured portfolio components

PROPOSED HEDGING TRANSACTION

- Trade date: 27/06/2004;
- Termination date: 31/12/2022;
- Notional: € 7.600.000;

FIXED AMOUNTS	FLOATING AMOUNTS
▪ Bank; ▪ 6% p.a.; ▪ act/360	▪ Municipality; ▪ act/360 ▪ Euribor 6 months Collar [2,8; 4] + step-up Spread:

07/2004 – 07/2006	1,50%
07/2006 – 07/2010	2,50%
07/2010 – 07/2017	3,00%
07/2017 – 12/2022	3,50%

STRUCTURED PORTFOLIO

Structured portfolio components

DEBT INSTRUMENT

- Trade date: 01/01/1998;
- Termination date: 31/12/2022;
- Notional: € 10.000.000;
- Fixed rate: 6% p.a.;
- Amortising: italian;
- Payment dates: semi-annually.

OTHER HEDGING TRANSACTION

NONE

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07/2004 – 07/2006	1,50%
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07/2010 – 07/2017	3,00%
07/2017 – 12/2022	3,50%

Computational & methodological steps

a) Parameters calibration

b) Stochastic processes numerical simulation

c) Probability distribution of the synthetic swap

d) Probabilistic valuation of the contract cost

Parameters should be determined on the basis of the market data at the proposal date and under the risk-neutral probability measure

Computational & methodological steps

a) Parameters calibration

b) Stochastic processes numerical simulation

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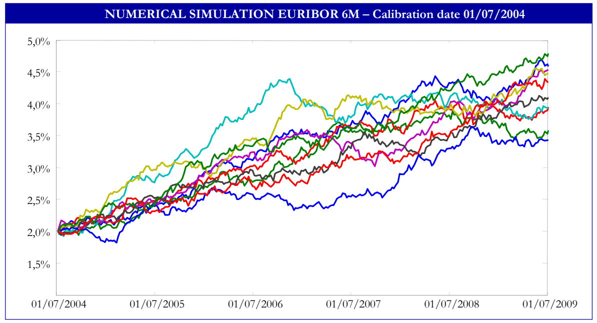
d) Probabilistic valuation of the contract cost



Computational & methodological steps

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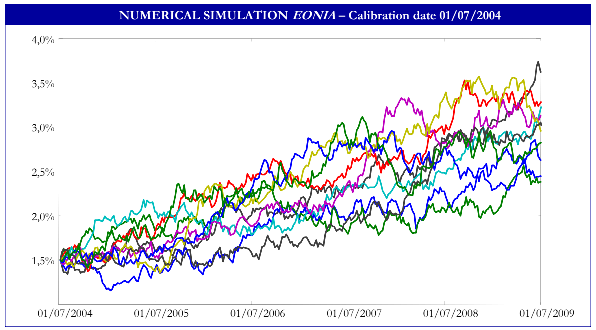

Valuation in 2004

Computational & methodological steps

a) Parameters calibration

b) Stochastic processes numerical simulation

c) Probability distribution of the synthetic swap

d) Probabilistic valuation of the contract cost

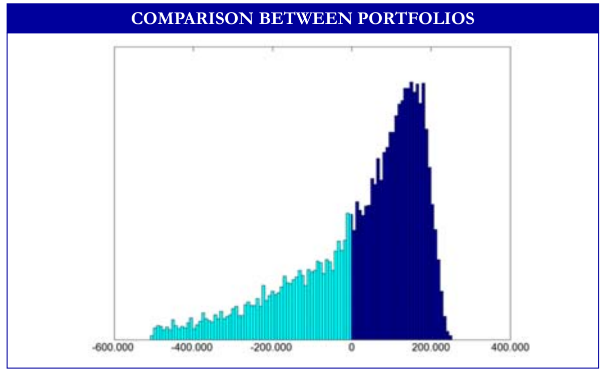

TABLE OF PROBABILISTIC PERFORMANCE SCENARIOS	PROBABILITY	AVERAGE VARIATION OF COSTS
The contract produces lower costs than the initial portfolio	65,7%	119.544 €
The contract produces higher costs than the initial portfolio	34,3%	-161.764 €

Valuation in 2004

Computational & methodological steps

a) Parameters calibration

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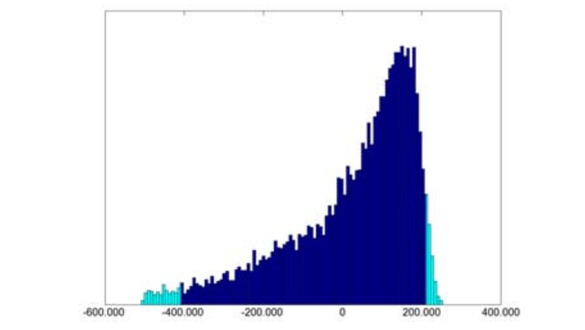
COMPARISON BETWEEN PORTFOLIOS

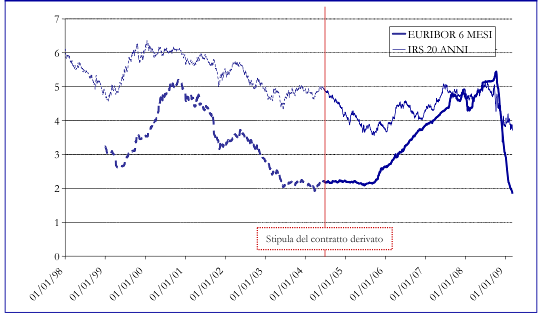


COHERENT PROBABILISTIC MEASURE	VALUES
"Minimum cost"	**-442.887€**
"Maximum cost"	**222.096 €**

FAIR VALUE = € - 23.168

Economic environment in 2009

INTEREST RATES 1998-2009

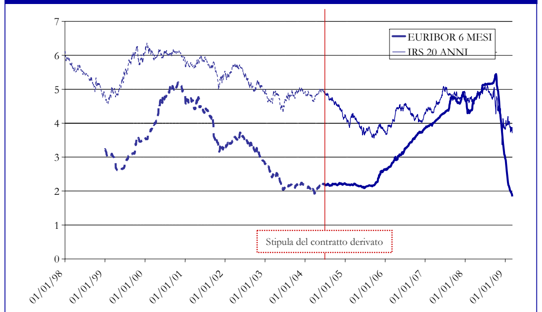


Municipality's decision problem

- **since the end of 2008 interest rates are strongly decreasing**
- **has the opportunity cost ratio of the structured portfolio changed?**
- **is it possible to benefit from the interest rate decrease by renegotiating the contract?**

Economic environment in 2009

INTEREST RATES 1998-2009



Municipality's decision problem

The valuation of the cost-opportunity of the structured portfolio should be compared with the initial portfolio, taking into account:

- **methodology utilised before trade;**
- **new market conditions;**
- **due cash flows (i.e. notional and interest).**

Computational & methodological steps

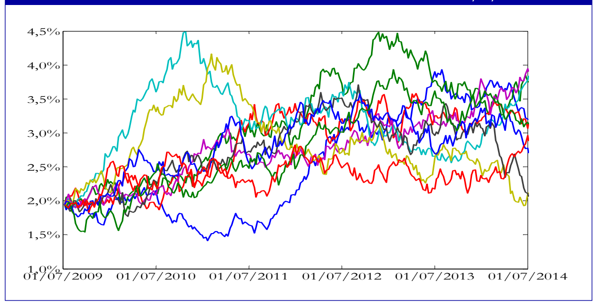
a) Parameters calibration

b) Stochastic processes numerical simulation

c) Probability distribution of the synthetic swap

d) Probabilistic valuation of the contract cost

NUMERICAL SIMULATION EURIBOR 6M – calibration date 01/03/2009



Computational & methodological steps

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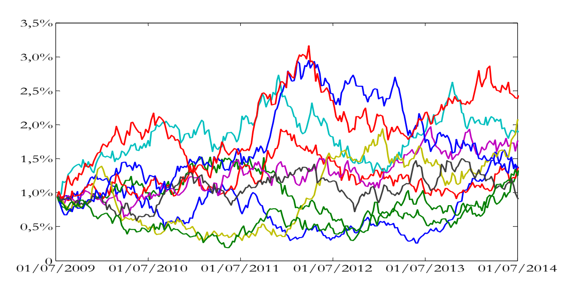
NUMERICAL SIMULATION *EONIA* – calibration date 01/03/2009



Valuation in 2009

Computational & methodological steps

a) Parameters calibration

b) Stochastic processes numerical simulation

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COMPARISON BETWEEN PORTFOLIOS

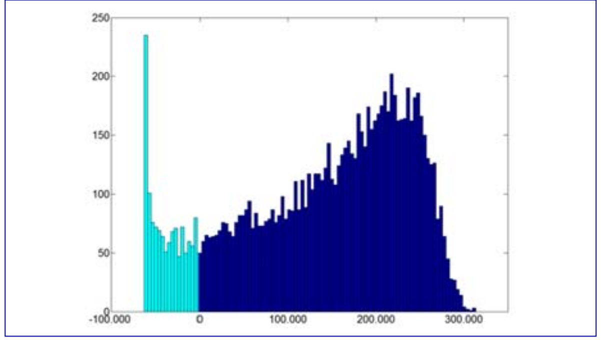


TABLE OF PROBABILISTIC PERFORMANCE SCENARIOS	PROBABILITY	AVERAGE VARIATION OF COSTS
The contract produces lower costs than the initial portfolio	12,7%	-36.131 €
The contract produces higher costs than the initial portfolio	87,3%	163.991 €

Valuation in 2009

Computational & methodological steps

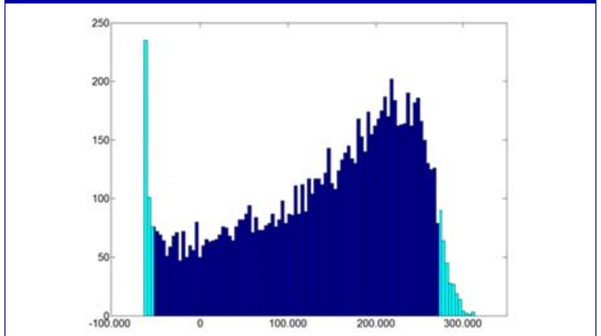
a) Parameters calibration

b) Stochastic processes numerical simulation

c) Probability distribution of the synthetic swap

d) Probabilistic valuation of the contract cost

COMPARISON BETWEEN PORTFOLIOS



COHERENT PROBABILISTIC MEASURE	VALUES
"Minimum cost"	**-60.229 €**
"Maximum cost"	**283.472 €**

FAIR VALUE = € - 138.636

Valuation in 2009

- Euribor 6 mesi Collar [2,8;4] + step-up Spread:

07/2004 – 07/2006	1,50%
07/2006 – 07/2010	2,50%
07/2010 – 07/2017	3,00%
07/2017 – 12/2022	3,50%

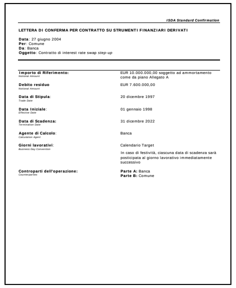
TABLE OF PROBABILISTIC PERFORMANCE SCENARIOS	PROBABILITY	
	2004	2009
The contract produces lower costs than the initial portfolio	37,6%	12,7%
The contract produces higher costs than the initial portfolio	62,4%	87,3%

COHERENT PROBABILISTIC MEASURE	VALUES IN EUR	
	2004	2009
"Minimum cost"	-616.600 €	-60.229 €
"Maximum cost"	646.788 €	283.472 €

The current market conditions affected the valuation of the Collar:

- **notwithstanding an high volatility environment, Municipality cannot benefit from the interest rate reduction because of the Collar Floor (2,8%);**
- **Municipality's position became worsen in terms of cost for re-negotiation.**

Term sheet example

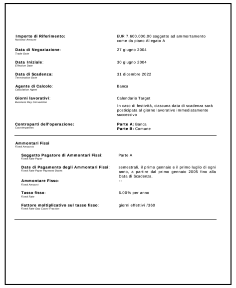


ISDA Standard Confirmation

LETTERA DI CONFERMA PER CONTRATTO SU STRUMENTI FINANZIARI DERIVATI

Data: 27 giugno 2004
Per: Comune
Da: Banca
Oggetto: Contratto di interest rate swap *step-up*

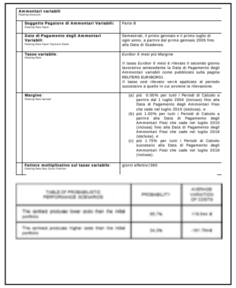
Importo di Riferimento: *Notional Amount*	EUR 10.000.000,00 soggetto ad ammortamento come da piano Allegato A
Debito residuo *Notional Amount*	EUR 7.600.000,00
Data di Stipula: *Trade Date*	20 dicembre 1997
Data Iniziale: *Effective Date*	01 gennaio 1998
Data di Scadenza: *Termination Date*	31 dicembre 2022
Agente di Calcolo: *Calculation Agent*	Banca
Giorni lavorativi: *Business Day Convention*	Calendario Target In caso di festività, ciascuna data di scadenza sarà posticipata al giorno lavorativo immediatamente successivo
Controparti dell'operazione: *Counterparties*	**Parte A:** Banca **Parte B:** Comune



Importo di Riferimento: *Notional Amount*	EUR 7.600.000,00 soggetto ad ammortamento come da piano Allegato A
Data di Negoziazione: *Trade Date*	27 giugno 2004
Data Iniziale: *Effective Date*	30 giugno 2004
Data di Scadenza: *Termination Date*	31 dicembre 2022
Agente di Calcolo: *Calculation Agent*	Banca
Giorni lavorativi: *Business Day Convention*	Calendario Target In caso di festività, ciascuna data di scadenza sarà posticipata al giorno lavorativo immediatamente successivo
Controparti dell'operazione: *Counterparties*	**Parte A:** Banca **Parte B:** Comune

Ammontari Fissi
Fixed Amounts

Soggetto Pagatore di Ammontari Fissi: *Fixed Rate Payer*	Parte A
Date di Pagamento degli Ammontari Fissi: *Fixed Rate Payer Payment Dates*	semestrali, il primo gennaio e il primo luglio di ogni anno, a partire dal primo gennaio 2005 fino alla Data di Scadenza.
Ammontare Fisso: *Fixed Amount*	--
Tasso fisso: *Fixed Rate*	6.00% per anno
Fattore moltiplicativo sul tasso fisso: *Fixed Rate Day Count Fraction*	giorni effettivi /360



Ammontari variabili
Floating Amounts

Soggetto Pagatore di Ammontari Variabili: *Floating Rate Payer*	Parte B
Date di Pagamento degli Ammontari Variabili: *Floating Rate Payer Payment Dates*	Semestrali, il primo gennaio e il primo luglio di ogni anno, a partire dal primo gennaio 2005 fino alla Data di Scadenza.
Tasso variabile: *Floating Rate*	*Euribor 6 mesi* più Margine Il tasso *Euribor 6 mesi* è rilevato il secondo giorno lavorativo antecedente la Data di Pagamento degli Ammontari variabili come pubblicato sulla pagina REUTERS EURIBOR01. Il tasso così rilevato verrà applicato al periodo successivo a quello in cui avviene la rilevazione.
Margine: *Floating Rate Spread*	(a) più 0.00% per tutti i Periodi di Calcolo a partire dal 1 luglio 2004 (incluso) fino alla Data di Pagamento degli Ammontari Fissi che cade nel luglio 2010 (esclusa), e (b) più 1.00% per tutti i Periodi di Calcolo a partire alla Data di Pagamento degli Ammontari Fissi che cade nel luglio 2010 (inclusa) fino alla Data di Pagamento degli Ammontari Fissi che cade nel luglio 2018 (esclusa), e (c) più 1.75% per tutti i Periodi di Calcolo successivi alla Data di Pagamento degli Ammontari Fissi che cade nel luglio 2018 (inclusa).
Fattore moltiplicativo sul tasso variabile: *Floating Rate Day Count Fraction*	giorni effettivi/360

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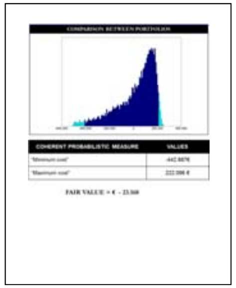


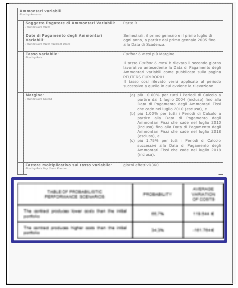
COMPARISON BETWEEN PORTFOLIOS

COHERENT PROBABILISTIC MEASURE	VALUES
"Minimum cost"	-442.887€
"Maximum cost"	222.398 €

FAIR VALUE = € - 23.168

FRONTIERS IN FINANCIAL MARKETS MATHEMATICS
Bologna, 13-17 LUGLIO 2009
Quantitative measures for an unified approach
to risks disclosure in financial products

Term sheet example



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