A supervisory perspective on Insider Trading: estimating the value of the information.

Marcello Minenna[¤]

26th September 2000

Abstract: the enforcement of the ban of the insider trading requires the evaluation of the disgorgement, i.e. the capital gain of the insider trader who takes advantage of the exploitation of preferential information. The ¯rst step forward on this topic has been made by the SEC, the United States Securities and Exchange Commission, that has developed a quantitative procedure based on the event study methodology. This paper develops an adaptation of this procedure for the Italian market and explains what the limits of this methodology are in the analysis of the insider trading phenomenon. Therefore, a new procedure, based on a probabilistic approach, to compute the economic value of the information exploited by the insider, has been developed. This methodology, which overcomes the issues connected with the event study procedure, o®ers extensive empirical evidence compared to the previous ones. Both procedures have been adopted by CONSOB: the Italian Securities and Exchange Commission, and have been presented to the Tribunal of Milan.

Key words: SEC, CONSOB, insider trading, disgorgement, event studies analysis, proxy variable, geometric Brownian motion, wiener process, abnormal return, cumulative abnormal return.

[¤]CONSOB, Intermediaries Division - Enforcement O±cer. The author expresses his gratitude to C.Milia and G.D'Agostino for the revision of the current work, to Professor G.Szego of University "la Sapienza" (Rome), and Professor S.Figlewski of New York University (NY) for the useful indications given. Moreover, the author thanks Professor J. Cvitanic of University of Southern California, Professor M. Smirnov and Dr. G. Petrella of Columbia University (NY) for the revision of the methodological aspects.

1 Introduction

All over the world, on a basis of the 103 countries that have stock markets, 87 of them present a regulation of the Insider Trading phenomenon (Bhattacharya, Daouk, 1999). This situation is the result of a dispute between the two main theoretical streams, which can be succinctly represented as follows: the ¯rst is convinced that the ban on insider trading would reduce market e±ciency and the managers' compensation, while the other states that the insider trader would appropriate the value of the preferential information[1] in fraud of the other investors and consequently the repression of this crime would increase the investors' trust in the market, and hence its liquidity. These theoretical streams have developed their arguments in more than 250 papers in the last forty years. These arguments can be summarized in three theories in favor of the insider trading repression and three more against it. (Bainbridge, 1988)

The three theories against the enforcement of the ban of insider trading can be de¯ned as follows:

1. victimless crime;
2. managers compensation;
3. market e±ciency;

The ¯rst one (Herzel and Katz, 1987[2]) states that insider trading has no victim; this is because transactions made by the insider would move the stock price in the same direction as the preferential information and consequently the counterpart of the insider would also take advantage of the insider transactions. For instance, in the case of bullish information the insider would raise the stock price and consequently the counterpart would sell the stock at a higher price than he would have without the insider transactions.

The second theory is based on the concept that the only e®ective way to compensate the managers is through the exploitation of preferential information. This is because of the fact that bonus and stock options are not °exible enough and ¯nancially viable for the company (Manne, 1966).

The last theory against the regulation of the crime exploits the concept of market e±ciency in its strong way, i.e. the stock price re°ects all the available information, also the preferential one. Hence, the insider by carrying out his strategy would push the stock price faster towards the value which would better re°ect the fundamentals of the company (Finnerty, 1976).

As far as the three theories which support the repression of insider trading are concerned, these can be de¯ned as follows:

1. misappropriation theory
2. market egalitarianism
3. market integrity

The ¯rst theory bases its main argument on the idea that preferential information is property of the company. Therefore, any exploitation of information

[1] It is reported in Appendix A a glossary of the terms used in the paper.

[2] The view that insider trading is a \victimless crime" is a popular one. Hertzel and Katz, in their paper explain this theory and criticize it.

carried out by a subject other than the owner, i.e. the company, could be assimilated to theft. (Georges 1976)

The market egalitarianism theory is based upon the argument that all the investors should take their investment decisions on the basis of the same information set, in order to have the same pay-off opportunities. (Loss 1983 Langevoort 1987)

Finally, the third theory stresses the concept of the integrity of the market. This theory argues that the insider trader damages the market and particularly to its micro-structure. This damage moves through two main channels that operate in a chain reaction: the operativity of the market makers and the operativity of the investors. In fact, the presence of insider traders in the market creates losses to the market makers that, in order to maintain long term profitability tend to increase the bid-ask spreads. This situation creates an increase in the transaction costs, which operates as a tax on all the investors, and creates a disincentive in the trading activity. These effects cause, firstly, a decrease in the liquidity of the market and in the signalling role played by the price, and secondarily a reduction of the market efficiency and finally an increase in the cost of capital for the companies[3] (King and Roell, 1988; Bhattacharya, Daouk, 1999).

The brief analysis of these theories and of their different arguments offers an easy explanation of why only some of the countries which have a stock market have regulated insider trading. A deeper analysis of the phenomenon, unfortunately, shows, that out of 87 countries, only 38 have really enforced this crime (Bhattacharya, Daouk, 1999).

This consideration highlights a new worrying perspective on this subject that cannot be restricted to the aforementioned theoretical dispute. In fact, the enforcement of the ban of insider trading presents a lot of operative issues for the supervisors.

Some quantitative procedures have to be used in order to detect the phenomenon, to compute the value of preferential information and hence, to calculate the disgorgement, that is the undue wealth gained by the insider through the exploitation of preferential information.

While the detecting phase of the insider affects the level of sensitivity in the market analysis carried out by the Supervisor, and hence the amount of signals that have to be put under examination, the evaluation of the disgorgement offers in all the legal systems punishing the crime of insider trading, a benchmark to identify the sanction to be imposed against the insider and in this way it can be considered as the link between financial and legal aspects.

Therefore the Supervisors, in the enforcement of the ban of this crime, have to be accurate in the identification of the value of the information the insider trader would appropriate (Mitchell, Netter, 1994).

Hence, the difficulty in identifying an objective, realistic and effective way to compute this value can create issues in assessing the damage caused by the insider to the market and, consequently to the enforcement action.

[3] This is also the main argument of the Regulators.

One of the most important contributions to this subject has been made by the SEC that developed in the 80s, the ¯rst quantitative methodology to compute the disgorgement by developing an econometric approach based on the event studies theory.

The purpose of this paper is ¯vefold. First, it sketches the legal references for the prosecution of insider trading in the United States of America and in Europe with a recapitulation scheme on the main peculiarities on how Italy, France, Germany and the United Kingdom have adopted the European directive. Secondly, it illustrates the methodologies used by the regulators, their limits and why the SEC has developed an econometric procedure. Thirdly, it wholly explains the rational behind this procedure and how it operatively develops. Fourthly, it shows an adaptation of this procedure to the Italian market and how it is operatively used. This solution has been recently adopted by CONSOB. Finally, it explains the limits of the econometric procedures and presents a new methodology to study the insider trading phenomenon that, unlike the event study theory, adopts a probabilistic approach. The paper shows the advantages of this procedure and why it theoretically overcomes the previous ones.

The new procedure has been adopted by CONSOB and it is showing substantial empirical results. The procedure has also been presented to the Tribunal of Milan[4] which deals with most of the insider trading cases.

2 The legal framework for the Insider trading repression

In this section, how the prosecution of the crime of insider trading occurs will be presented. To achieve this according to the di®erent legal frameworks of various countries, the scope of the rules, the de¯nition of preferential information, the subjects under supervision and what type of behavior is forbidden will be illustrated.

In general terms, two main rules have to be taken into account in the repression of the crime:

1. home country control: every country is in charge of the monitoring of insider trading on the stocks quoted in the stock exchange established in its territory.

2. co-operation between the Authority in charge of the Insider Trading control and the Judicial system.

2.1 The legal establishment in the United States of America

In the United States the ¯rst prosecution of insider trading under State law, occurred in 1903 . Despite this occurrence, the legal establishment for the

[4] In the Italian Judicial system the Tribunal is the ¯rst instance Court.

repression of insider trading was introduced with section 16 of the Securities Exchange Act in 1934. This law did not take into account the use of preferential information made by the insider and imposed the prohibitions only on directors, o±cers and those shareholders having more than 10% of the registered capital. Due to these objective and subjective limitations, SEC adopted the proxi considered in section 10 (b) of the above-mentioned act in order to enact the rules that protect the stock exchange from fraud. Hence, SEC enacted the rule 10b-5 in 1942. This rule, following section 17 of the Securities Act of 1933, removed the subjective limitations set out in section 16 and ¯lled in a clear loophole in the law, introducing the case of acquisition of securities, not included in section 17 of the Securities Act of 1933 (Loss 1970). Since 1942, the rules adopted to punish the crime of insider trading in the United States and the outline of the discipline have been a®ected by several interpretations given by SEC and the law. At the end of the 70s, the Second Circuit Court of Appeal pointed out an evident limitation of this discipline. Indeed, the continuous series of laws enacted between 1942 and 1980 required a ¯duciary duty between the seller and the counterpart in order to contemplate the crime of insider trading. As huge amounts of take overs, mergers and acquisitions, took place in the USA, at the end of the 70s, this law proved to be inadequate. On October 14th 1980, SEC, empowered by section 14-e of the Securities Exchange Act of 1934, enacted rule 14e in order to remove this additional subjective restriction.

The rules mentioned above coupled with several decisions made by the District of Columbia, the Second and Ninth circuit Courts of ¯rst instance and the Second, Fourth, Eighth circuit Courts of Appeal and the Supreme Court represent the legal framework of reference in the United States[5].

The basic elements of the discipline are the following:

Scope

The prohibition is imposed on the purchasing and selling related to those securities listed on one of the national Stock Exchanges and carried out on the market or in transactions carried out by individuals without ¯nancial intermediaries and outside the regulated markets (i.e. face-to-face transactions).

Preferential information

The prohibition is imposed on material and non-public information. In compliance with what the Supreme Court sets out, a piece of information has to be considered as material when a \reasonable" investor sees its disclosure as of paramount importance for an investment. When a corporate (information related to the issuing body) or a market information (information related to the whole market or to the sector in which the issuing body works) is kept secret it is considered as non public.

Prohibitions are imposed on the following subjects

Prohibitions are imposed on:

A) every subject that knows preferential information;

B) every subject having ¯duciary duty towards the owner of the information

[5] A good recapitulation of these decisions is in Loss (1983), Langevoort (1987), in Georges (1976), in Kraakman (1991), in Hagen (1988), in Martin (1986) and in Bergmans (1991).

(misappropriation theory);

C) every subject that receives information that is non public (\tippee")

Forbidden behavior

Insiders are not allowed:

- to carry out ¯nancial transactions when they consciously have preferential information (prohibition of trading);

- to provide third parties with this information (tipping);

- to suggest that a third party should carry out transactions in the market based on this information (tuyautage);

- to prompt a third party to carry out transactions.

As far as the role of the SEC and of the Judicial System in the repression of this crime in the USA are concerned, the SEC is empowered of civil actions and interacts with the Judicial System for penal action.

2.2 The legal establishment in Europe

In Europe the regulation of insider trading is dealt with within the EEC directive 89/592 (November 13th, 1989) and its basic elements are the following:

Scope

Article 1 states that the law can only be enforced on ¯nancial transactions carried out on a market which is \regulated and supervised by authorities recognized by public bodies" and which \operates regularly and is accessible directly or indirectly to the public.

Paragraph 3 of the article 2 states that the law has to be enforced only on those transactions taking place with the intervention of a professional intermediary. Each member state is empowered to enforce the law on those transactions carried out by individuals without ¯nancial intermediaries and outside the regulated markets (face-to-face transactions).

Article 5 de¯nes the territorial jurisdiction: each member state is entitled to enforce the prohibitions \at least to actions undertaken within its territory to the extent that the transferable securities concerned are admitted to trading on a market of a Member State". Anyway, each member state has to take in those transactions related to real values carried out inside a regulated market \situated or operating within that territory".

Preferential information

Article 1 n.1 of the directive de¯nes a piece of inside information as \information which has not been made public of a precise nature relating to one or several issuers of transferable securities or to one of several transferable securities, which, if it were made public, would be likely to have a signi¯cant e®ect on the price of the transferable security or securities in question".

Hence, preferential information may consist of peculiar corporate information related to the issuing body (corporate information) or of general information related to the whole market or to the sector in which the issuing body works (market information).

Moreover, preferential information is to be kept secret. In compliance with the ongoing law, this kind of information is no longer considered to be preferential when it is accessible to other parties even though they do not actually know of it.

Prohibitions are imposed on the following subjects

The prohibitions set out by the directive are imposed on the following subjects:

A) institutional insiders: those \by virtue of [their] membership of the administrative, management or supervisory bodies of the issuer" who have preferential information (art. 2, n.1);

B) other primary insiders: those that have access to the information \by virtue of the exercise of their employment, profession or duties" (art. 2, n.1);

C) \tippee": as for article 4, he is \any person other than those referred to" in the art. 2 (basic or institutional insiders) who \with full knowledge of the facts possesses inside information, the direct or indirect source of which could not be other than a person referred to in art.2".

Forbidden behavior

The subjects mentioned in A and B are not allowed:

- to buy or sell, on their account or on behalf of a third party, directly or indirectly, those real values related to the preferential information (prohibition of trading) deliberately using the information;

- to provide third parties with preferential information \unless such disclosure is made in the normal course of the exercise of his employment, profession or duties" (tipping);

- to suggest that a third party should carry out transactions related to the real values that the preferential information is about (tuyautage);

- to prompt a third party to carry out transactions.

The subject mentioned in C is not allowed to:

- trade, even though each member state can also impose the prohibition of tipping and tuyautage on this subject, which are usually imposed on the institutional or basic insiders.

As far as the role of the Authority, empowered of the Insider Trading control and of the Judicial system in the repression of the crime is concerned in the EEC, table 1 gives a comprehensive explanation of how Italy, France, UK and Germany has adopted the European Directive.

	Italy	Germany	United Kingdom	France	
Law Reference	Consolidated Act on Financial Intermediation (1998)	Securities Trading Act (1994)	Criminal Justice act (1993)	COB Rule 90-08 (1991)	Ordonnance 67-833 (1967)
Scope	Any Securities listed on a Domestic or EEC stock exchange	Any Securities listed on a Domestic or EEC stock exchange	Any Securities listed on a Domestic or EEC stock exchange	Any Securities listed on a Domestic stock exchange	Any Securities listed on a Domestic stock exchange
	Also Face -to - Face Transactions	Also Face -to - Face Transactions		Also Face -to - Face Transactions	Also Face -to - Face Transactions
Inside Information	non Public, Precise and Significantly Price Sensitive	non Public and Price Sensitive	non Public, Precise and Significantly Price Sensitive	non Public, Precise and Price Sensitive	non Public and Price Sensitive
Subjects Enforced	institutional insiders	institutional insiders	institutional insiders	institutional insiders	institutional insiders
	Other Primary Insiders	Other Primary Insiders	Other Primary Insiders	Other Primary Insiders	Other Primary Insiders
	Tippees	Tippees*	Tippees	Tippees*	
Forbidden Behaviour	Trading	Trading*	Trading	Trading*	Trading
	-on own account	-on own account	-on own account	-on own account	-on own account
	-On behalf of third parties	-On behalf of third parties	-On behalf of third parties	-On behalf of third parties	-On behalf of third parties
	Suggest the preferential information to third parties	Encourage third parties to exploit the preferential information	Encourage third parties to exploit the preferential information	Encourage third parties to exploit the preferential information	Encourage third parties to exploit the preferential information
	Communicate the Information to third parties	Communicate the Information to third parties	Communicate the Information to third parties	Communicate the Information to third parties	Communicate the Information to third parties
Sanctions					
Penal Sanctions	Imprisonment	Imprisonment	Imprisonment		Imprisonment
	Fine: (also) in relation to the Disgorgement	Fine: in relation to the Disgorgement	Fine: no limit (also in relation to the Disgorgment)		Fine: in relation to the Disgorgement
Subjects in Charge of Carrying out the Sanctions	Judicial System	Judicial System	Judicial System		Judicial System
Civil Sanctions			Fine: in relation to the Disgorgement		
			Inprisonment*		
Subjects in Charge of Carrying out the Sanctions			FSA, LSE, DTI, Bank of England, BSC, CPS*		
Administrative Sanctions		Suspend the Authorization of providing Financial Services.		Fine: in relation to the Disgorgement	
Subjects in Charge of Carrying out the Sanctions		BAEW		COB, CBV, CMT	

BAEW: The Germany Securities and Exchange Commission

FSA: The Financial Service Authority

LSE: The London Stock Exchange

DTI: Department of Trade nad Industry

BSC: The Building Society Commission

CPS: The Crown Prosecution Seervice

COB: The French Securities and Exchange Commission

CBV, CMT: the French Security Exchange

3 The computation of the disgorgement

As already stated, the laws prohibiting insider trading identify in the disgorgement a benchmark to quantify the sanctions against the insider[6]. Therefore careful evaluation is necessary and has to be applied to all cases provided for and accepted by the Judicial power in those countries where the legal system empower them to impose similar sanctions.

The ¯rst method of evaluation, adopted by the Supervisors, consists in calculating the actual capital gain. In this case, the disgorgement corresponds to the di®erence between the closing value of the insider position over the security (usually after the disclosure of the preferential information) and the opening value of the position. However, this method is not e®ective if the insider closes the position a long time after the disclosure of the information or if the position is not closed at all; in fact in this case, the connection between the news and the insider position vanishes.

In order to overcome these di±culties, the supervisors generally calculate the disgorgement as the di®erence between the price after the disclosure of the information and the opening price of the position, multiplied by the invested quantities. This methodology is de¯ned as potential deterministic disgorgement.

Yet, this procedure could cause some problems too; for example, if the insider opens the position a long time before the disclosure of the information its pro¯tability may be a®ected by events unrelated to its trading.

In order to tackle all these problems, SEC has developed a procedure on the basis of the Event Study Analysis allowing the determination of the price percentage variation of the security caused by the preferential information. This computation is founded on the relation between the pro¯t obtained by the security and the market pro¯tability. This is de¯ned potential econometric disgorgement. This paper shows an adjustment to the Italian market of this methodology developed in the US, that has been adopted at CONSOB.

The potential econometric disgorgement method has improved the procedure related to the evaluation of the pro¯t gained by the potential insiders. Yet, this method causes some di±culties such as the individuation of a market proxy portfolio statistically robust, the need for a long historical time series data set and the condition that a linear deterministic relation found out in the past is stable and e®ective also in the future.

To overcome these issues a new methodology to study the insider trading phenomena based on a probabilistic approach has been developed and it is currently used inside CONSOB. The procedure leads to the computation of the potential probabilistic disgorgement, analyzing all the future price scenarios, giving them a suitable probability, on the basis of the strategy of the insider and on the current stock price.

In order to better understand how the potential econometric digorgement is operatively calculated both in the US and in the Italian methodologies, and

[6] It is important to highlight that it is not equal in the di®erent legislations the importance of the disgorgement and its role in the determination of the sanctions against the insider.

what are the problems related to this computation which have conducted to a new probabilistic approach, it is necessary to analyze how and why the Event Study Analysis has traditionally been developed.

3.1 Potential Econometric Disgorgement

3.1.1 Event Study: traditional approach

The evaluation of the impact of an event on the value of a company is a di±cult task for economists. The event study analysis, which estimates the e®ect on stock returns of occurrences, such as mergers, acquisitions, take-overs, announcements, variation of the regulation in the reference microeconomical system, etc., is widely used. The ¯rst publication concerning the event study methodology dates back to 1933 (Dolley). Over the years this methodology has been applied in a variety of ¯elds, such as the study of Insider Trading phenomena[7].

The traditional methodology consists of nine fundamental steps:

1. the de¯nition of the events to be studied and the reference time horizons for the analysis.

Supposing that the date of the event is ¡, there is a time horizon used for the estimate of the model parameter $® = T_0 \; ! \; T_1$ that is de¯ned before the event, a time horizon which contains the event $£ = T_{1+1} \; ! \; T_2$ for a veri¯cation of the signi¯cance of the regression model de¯ned in the period ® and consequently for the estimate of the e®ect on stock return of the event just highlighted (¯gure 1);

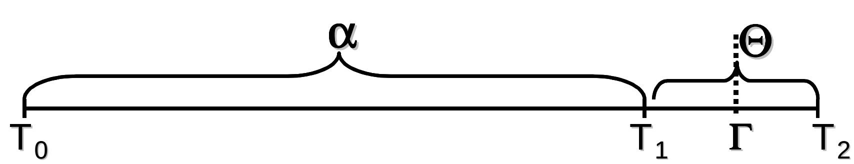

Figure 1:

2. analysis of the company history in the reference time horizon in order to detect the variations of the company stocks value; to avoid the presence of breaks in the series of stock returns due to information heterogeneity.

Returns[8] are de¯ned as:

$\ln \frac{S_t}{S_{t \text{ ¡ } 1}}$ where S_t is the value of the stock in time t:

[7] For an exaustive discussion of the increasing level of sophistication of the event study over the decades see Copeland and Weston, chapter 4 of \Financial Theory and Corporate Policy" (1992), Myers and Bakay (1948), Fama, Fisher, Jensen and Roll (1969).

[8] The dividends can be included or not in the analysis, just by ¯tting the de¯nition of the stock return.

This is done because it is hypothesized that the return $\frac{S_t}{S_{t-1}}$ has a lognormal distribution, therefore the logarithm of this random variable is distributed as a normal:

$$\ln \frac{S_t}{S_{t-1}} \sim N(\mu; \sigma^2)$$

3. determination of parameters to be employed for the assessment of the normal and, as a consequence, of the abnormal return. A widely employed statistical model is the Market model which explains the relationship between the returns of the i^{th} firm and the market portfolio through the linear regression model:

$$R_{it} = \beta_{0i} + \beta_{1i} R_{mt} + \varepsilon_{it}$$

that is:

$$E(R_{it}) = \beta_{0i} + \beta_{1i} E(R_{mt})$$

where the model hypothesizes that

i. the regressor observations are independent
ii. $\varepsilon_i \sim N(0; V_i)$
iii. ε_i are independent random variables; this means that there is not a serial correlation between the errors:
$Cov(\varepsilon_{it}; \varepsilon_{i\tau}) = 0$ where $t \neq \tau$

In graphic terms the simple linear regression model can be represented as shown in figure 2:

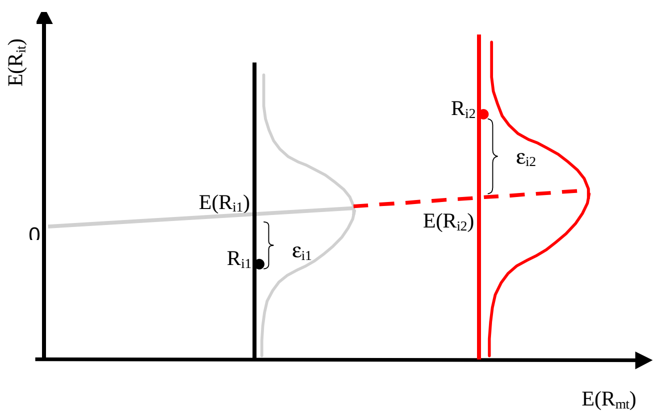

Figure 2:

Generally speaking, in the whole time horizon α the model can be rewritten for the i^{th} stock as follows:

$$\begin{bmatrix} R_{iT_0+1} \\ R_{iT_0+2} \\ \vdots \\ \vdots \\ R_{iT_1} \end{bmatrix} = \begin{bmatrix} 1 & R_{mT_0+1} \\ 1 & R_{mT_0+2} \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & R_{mT_1} \end{bmatrix} \cdot \begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{iT_0+1} \\ \varepsilon_{iT_0+2} \\ \vdots \\ \vdots \\ \varepsilon_{iT_1} \end{bmatrix}$$

that is

$$\underset{\alpha \times 1}{R_i} = \underset{\alpha \times 2}{R_m}\underset{2 \times 1}{\beta} + \underset{\alpha \times 1}{\varepsilon_i}$$

4. obviously, the estimate of parameters takes place for every i^{th} stock with the Ordinary Least Squared (OLS) method in the period that is:

$$\min_{\beta} \sum_{t \in \alpha} (R_{it} - E(R_{it}))^2$$

that in matrix notation equals:

$$\min_{\beta} (\underset{\alpha \times 1}{R_i} - \underset{\alpha \times 2}{R_m}\underset{2 \times 1}{\beta})' \cdot (\underset{\alpha \times 1}{R_i} - \underset{\alpha \times 2}{R_m}\underset{2 \times 1}{\beta})$$

The result of this minimization leads to the identification of estimators $\hat{\beta}_{0i}$; $\hat{\beta}_{1i}$ for parameters β_{0i} and β_{1i}:

$$\begin{bmatrix} \hat{\beta}_{0i} \\ \hat{\beta}_{1i} \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 1 & \cdots & 1 \\ R_{mT_{0+1}} & R_{mT_{0+2}} & \cdots & R_{mT_1} \end{bmatrix} \cdot \begin{bmatrix} 1 & R_{mT_{0+1}} \\ 1 & R_{mT_{0+2}} \\ \vdots & \vdots \\ 1 & R_{mT_1} \end{bmatrix} \right\}^{-1} \cdot \begin{bmatrix} 1 & 1 & \cdots & 1 \\ R_{mT_{0+1}} & R_{mT_{0+2}} & \cdots & R_{mT_1} \end{bmatrix} \cdot \begin{bmatrix} R_{iT_{0+1}} \\ R_{iT_{0+2}} \\ \vdots \\ R_{iT_1} \end{bmatrix}$$

where the estimator $\underset{2 \times 1}{\hat{b}}$ is consistent by construction: $E(\underset{2 \times 1}{\hat{b}}) = \underset{2 \times 1}{\beta}$. The identification of these parameters is necessary for the definition of the regression line for the single i^{th} stock where the i^{th} return is generally called fitted and the market return regressor, as shown in figure 3:

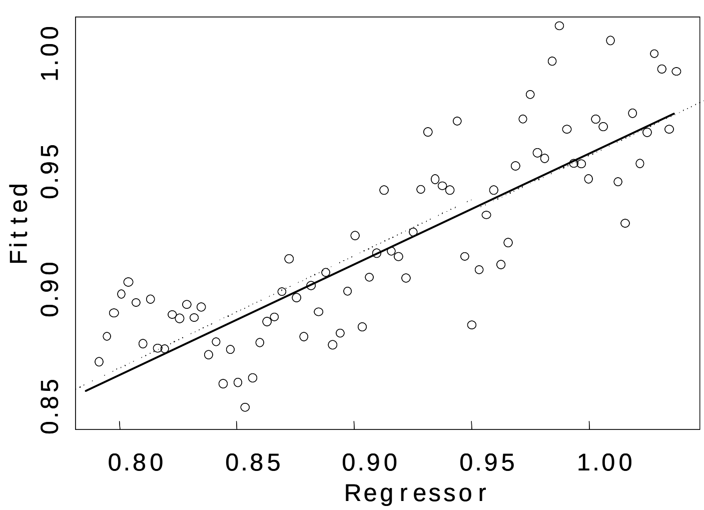

Figure 3:

5. therefore these parameters $\overrightarrow{\hat{\beta}}$ are employed in the time horizon τ as indicators of the normal return and so as a statistical basis to identify the abnormal return. In fact, employing the regression model:

$$\underset{\tau\times 1}{R_i} = \underset{\tau\times 2}{R_m}\,\underset{2\times 1}{\hat{\beta}} + \underset{\tau\times 1}{e_i}$$

therefore the estimation error $\overrightarrow{\hat{e}_i}$ represents the error in the normal return estimation given by the regression model. This error is defined as potential abnormal return (AR) and can be identified as the estimation error in the prediction during the period τ based on regression parameters determined in the horizon α:

$$\overrightarrow{\hat{e}_i} = \overrightarrow{\hat{AR}_i} = \overrightarrow{R_i} - R_m\overrightarrow{\hat{\beta}}$$

For the hypotheses of the model the potential abnormal return distribution is normal with the following parameters:

$$\overrightarrow{\hat{AR}_i} \sim N(0; V_i)$$ [9]

where

$$V_i = I\cdot\sigma_i^2 + E\left(R_m(\hat{\beta}-\beta)(\hat{\beta}-\beta)' R_m' \mid R_m\right)$$

The second term shows a dependence on the market returns vector, breaking, in this way, the hypothesis of independence of the regressor observations. It is important to note that as the length of period α increases this serial correlation vanishes because, when this estimation interval increases, the term $(\hat{\beta}-\beta)$ is frustrated.

6. construction of a statistic in order to verify more easily the level of abnormality expressed by the r.v. AR in the period τ, compared to the model built in the period α: By using, the statistical distribution of the i^{th} AR, and by defining $\overrightarrow{S_i^0}$ as the vector of the standard deviation of the $\overrightarrow{\hat{AR}}$, i.e.[10]:

$$\overrightarrow{S_i^0}(\tau) = \left(\sigma_{i_{T_1+1}}\quad \sigma_{i_{T_1+2}}\quad \ldots\quad \sigma_{i_{T_2-1}}\quad \sigma_{i_{T_2}}\right)$$

it is possible to define the r.v. SAR, Standardized Potential Abnormal Return, that will be distributed by construction as follows:

$$\overrightarrow{S\hat{AR}_i}(\tau) = \frac{\overrightarrow{\hat{AR}_i}(\tau)}{S_i^0} \sim N(0;1)$$

In order to define the distribution of the SAR, the ignorance of the value σ_i calls for the employment of an estimator. The estimator to be employed is simply the standard deviation estimator connected with the fitted prediction in the period τ; employing the parameters determined in the period α. The single element of the vector $\overrightarrow{\hat{S}_i}$ is determined as follows:

$$\hat{\sigma}_{i_{T_1+1}} = \sqrt{\frac{\sum_{\alpha} \hat{e}_i^2}{n-2}\left(1+\frac{1}{N}+\frac{\left(\overline{R_m}-R_{m_{T_1+1}}\right)^2}{\sum_{\alpha}\left(R_{tm}-\overline{R_m}\right)^2}\right)}$$

By defining $\overrightarrow{\hat{S}_i^0}$ the vector of the estimated standard deviation of the i^{th} r.v. AR, i.e.:

$$\overrightarrow{\hat{S}_i^0} = \left(\hat{\sigma}_{i_{T_1+1}}\quad \hat{\sigma}_{i_{T_1+2}}\quad \ldots\quad \hat{\sigma}_{i_{T_2-1}}\quad \hat{\sigma}_{i_{T_2}}\right)$$

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[9] The demonstration of the values assumed by the mean and the variance of r.v. AR has been shown in Appendix B.

[10] The underlying hypothesis is that the second term of V_i vanishes.

it clearly emerges that the distribution of SAR becomes t-student with $\alpha - 2$ degree of freedom:

$$\widehat{SAR}_i = \frac{\widehat{AR}_i}{\widehat{S}'_i} \sim \underset{df = \alpha - 2}{t_{student}}$$

This statistic is built on the basis of the residuals of the regression line expressed by the market model. Since by construction $E(\widehat{SAR}_i(\Theta)) = 0$, if in the period Θ this hypothesis is not verified, the model defined in α will not explain the return of the i^{th} stock in the period Θ and, therefore the potential abnormal returns will be effectively abnormal ones.

7. aggregation on N stocks of the i^{th} SAR. It is simply done by exploiting the SAR's distribution properties[11]. Indeed, it is sufficient to work on the r.v. average of the N SAR_i i.e.: $\overline{SAR}$

$$\overline{SAR} = \frac{1}{N} \sum_{i=1}^{N} SAR_i$$

Moreover, since the sum of random variables normally distributed is still distributed normally and the mean of the sum equals the sum of the means, the r.v. $\overline{SAR}$ is normally distributed too and $E(\overline{SAR}) = 0$

Hence, it follows that:

$$\sqrt{N} \cdot \overline{SAR} \sim \underset{df = \alpha - 2}{t_{student}}$$

8. hypotheses testing on the $\overline{SAR}$ statistic in order to verify if the event occurrence has determined an abnormal return in the period Θ: Since, as explained above, this statistic entails the property of the model defined in the period α, the violation of its distribution property, i.e. $E(\overline{SAR}) = 0$ will coincide with the rejection of the model in the period Θ and therefore with the conclusion that the events occurred in the period Θ have determined an abnormality level in the return of the analyzed stocks.

The test is then constructed as follows:

$H_0 : E(\overline{SAR}) = 0 \Rightarrow$ the events do not determine abnormal returns;

$H_A : E(\overline{SAR}) \neq 0 \Rightarrow$ the events determine abnormal returns;

The null hypothesis H_0 will be rejected if $\left| E(\overline{SAR}(\Theta)) \right| > t_{\frac{\imath}{2}}$ for some prescribed $\imath$. $\imath$ is defined as the significance level in hypothesis testing problems. It represents the Type I error accepted in the test, that is the probability of rejecting H_0 when this hypothesis is true.

It is easy to compute the power of the test also called p-value that is:

$p = P(E(\overline{SAR}) \neq 0 \mid H_0 \text{ is true})$

If $p < \imath$ the null hypothesis is true at that significance level $\imath$, viceversa if the null hypothesis is rejected, this will confirm the presence of an abnormal return in the period Θ.

9. calculation of the cumulative abnormal returns, in order to wholly represent the abnormality of the return over the period of analysis. To this end, it will be defined as the r.v. CAR (i.e. Cumulative Abnormal Return) given by the sum of the potential abnormal returns observed in the period Θ :

[11] The procedure shown hypothesizes that the stocks are non-correlated and that the event windows are not superimposed. Some straight forward computational adjustments are required to remove these hypotheses.

$$CAR_i = \sum_{j \in \Theta} AR_{ij}$$

The CAR distribution is by construction:

$$CAR_i(\Theta) \sim N(0; V_i)$$[12]

The aggregation on N stocks is particularly simple by exploiting the CAR's distribution properties[13]. It is sufficient to work on the average $\overline{CAR}$ statistics of the N $CAR_i(\Theta)$;

$$\overline{CAR}(\Theta) = \frac{1}{N} \sum_{i=1}^{N} CAR_i(\Theta)$$

The $\overline{CAR}$ distribution is for construction:

$$\overline{CAR}(\Theta) \sim N(0; \overline{V})$$

where:

$$\overline{V} = \frac{1}{N^2} \sum_{i=1}^{N} V_i$$

The graphical observation of this r.v., with respect to time, offers a clear and straight forward test of the abnormality of the returns over the period Θ: (figure 4.)

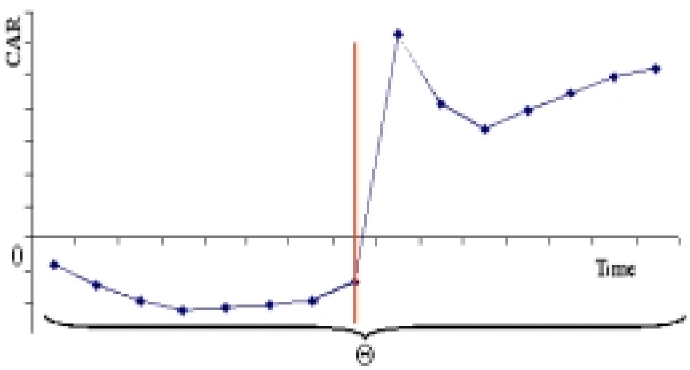

Figure 4:

In fact, it is easy to observe that the line that represents the $\overline{CAR}$, after the event occurred, moment indicated with the vertical line, moves far from the zero value increasing over time according to the value of the event and to its impact on the stock return.

Once, it has been clarified how and why the event study is able to capture the economic value of the information of a company occurrence, in order to compute the disgorgement, it is important to understand how operatively this theory has been used in US and in Italy

[12] Also in this case, the lack of a precise determination of V_i; as already underlined, entails the employment of the estimator $\hat{V}_i$ substantial by construction.

[13] Also for this r.v. it is fundamental to hypothesize that the stocks are non-correlated and that the event windows are not superimposed.

3.1.2 SEC methodology

The methodology described in the previous paragraph is applied with some simplifications by the SEC in order to analyze the Insider Trading phenomena and specifically to calculate the disgorgement.

In particular, the methodology employed by SEC corresponds to the methodology explained before, without the aggregation of the different stocks, since the Insider Trading research is carried out on the single case[14]. A short description follows making explicit the modalities for the disgorgement calculation.

1. Individuation of the insider event Γ and of a $\Theta = 20$ days and a $\alpha = 120$ days[15].

2. Analysis of the company price evolution in the observation period in order to standardize data.

3. Calculation of the stock returns in α as explained in point 2 of the previous paragraph.

4. Market model in the period α on the stock object of study:

$$\underset{\alpha\times1}{R_i} = \underset{\alpha\times2}{R_m}\,\underset{2\times1}{\beta} + \underset{\alpha\times1}{\varepsilon_i}$$

5. Estimate of the vector $\underset{2\times1}{\beta}$ with the least squared method.

6. Calculation of the potential abnormal return (AR) on Θ as previously explained, that is:

$$\widehat{\varepsilon}_i = \widehat{AR}_i = \widehat{R}_i - R_m\widehat{\beta}$$

7. Construction of the SAR statistic as explained in point 6 of the previous paragraph.

8. hypotheses testing on the SAR statistic over the period Θ; in order to verify if the disclosure of the preferential information has determined some abnormality in the returns of the stock under investigation.

Since, as explained before, this statistic entails the property of the model defined in the period α, the violation of its distribution property, i.e. $E(SAR) = 0$; will coincide with the rejection of the model in period Θ and therefore with the conclusion that the disclosure of the inside information occurred in period Θ have determined an abnormality level in the return of the investigated stock.

The test is then defined as follows:

$H_0 : E(SAR) = 0 \Rightarrow$ the inside information does not determine abnormal returns;

$H_A : E(SAR) \neq 0 \Rightarrow$ the inside information determines abnormal returns;

As in the previous paragraph, the significance level – to test the hypotheses will be defined, the power of the test, also called p-value, and the cumulative abnormal return in order to graphically represent the abnormality level of the stock return analyzed can be computed.

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[14] This simplification is not trivial from a statistical point of view, since it could create some convergency issues in the probability distribution of the stock return. Particularly, as better explained in the paragraph which explains the Consob Methodology, this choice combined with some market issues could violate *a priori* the statistical properties entailed by the model and hence it could render the disgorgement calculation meaningfulness.

[15] As seen in the previous paragraph, Θ is a time period that crosses the insider event, while α is defined before the preferential information is disclosured to the market.

9. the computation of the disgorgement consists in simply multiplying the abnormal return by the quantity involved in the insider transactions. Obviously, this computation will proceed if and only if the hypothesis testing confirms that the preferential information has determined an abnormality in the return.

3.1.3 CO.N.SO.B. methodology

The SEC methodology cannot be applied as it is to the Italian market. This is because of the peculiarities of the Italian market, such as:

i. the lack of weight of most listed stocks;

ii. the large presence of companies recently listed on the stock exchange;

iii. the observation of some seasonality effects.

The methodology can be exemplified in the following fundamental passages:

1. Individuation of the insider event $-$ defined as $-_2$:

2. The definition of α and β: As regard β it is fixed, as in the US procedure, equal to 20 days. Because of the Italian Market features, the choice of α has to be made with specific accuracy. For instance, the thickness of the quoted stocks implies that 120 observations would not be enough to ensure a statistical significance of the model and the presence of seasonality in market trends implies that the enlargement of the time window could include non-homogeneous observations.

An explanation of the solution which was developed in order to overcome these issues follows and that has been adopted by CONSOB.

The time horizon α is defined around 600 days[16]. This choice is supported by some empirical analyses of the Italian Stock Market Index (i.e. MIB) returns in connection with their convergence in distribution towards the normal driven by the central limit theorem. In these analyses the returns of n days with n variables have been aggregated in order to determine the value which leads to a random variable normally distributed with parameters μ and σ. This can be represented in formulae as follows[17]:

$$\text{Find } n^* \to P\left\{\frac{\ln\frac{S_1}{S_0}+\ln\frac{S_2}{S_1}+\ln\frac{S_3}{S_2}+\ldots+\ln\frac{S_n}{S_{n-1}}-n\mu}{\sigma\sqrt{n}} \le a\right\} \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{x^2}{2}}dx$$

So far, the procedure has overcome the first two problems related to the Italian market. As far as the seasonality phenomena observed on stocks listed in the Italian financial market is concerned, the methodology proceeds on broken single time windows rather than directly on one wider single time window. Particularly, it identifies the days $-_1$; $-_0$; as the same dates of the event $-_2$ in the two previous years. Hence, α becomes a vector: $\vec{\alpha} = (\alpha_0; \alpha_1; \alpha_2)$ where $\alpha_0; \alpha_1; \alpha_2$ equal to 200 days each are the time windows before $-_2$; $-_1$; $-_0$. Eventually, this vector defines 3 periods for a total of 600 observations.[18]

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[16] The choice of such a wide α makes it more suitable to analyse if there has been some extraordinary events for the company which could have generated some issues in the data set considered.

[17] The results of these empirical analyses are available from the author on request.

[18] The proposed solution has shown robust empirical evidence. The results of these empirical analysis are available from the author on request.

3. Market model on the stock object of study by using as regressors the MIB index, the Italian and the European sector indexes.

a) $\underset{\alpha\mathrm{x}1}{R_i} = \underset{\alpha\mathrm{x}2}{R_{MIB}}\ \underset{2\mathrm{x}1}{\beta} + \underset{\alpha\mathrm{x}1}{\varepsilon_i}$

b) $\underset{\alpha\mathrm{x}1}{R_i} = \underset{\alpha\mathrm{x}2}{R_{\text{Italian sector index}}}\ \underset{2\mathrm{x}1}{\beta} + \underset{\alpha\mathrm{x}1}{\varepsilon_i}$

c) $\underset{\alpha\mathrm{x}1}{R_i} = \underset{\alpha\mathrm{x}2}{R_{\text{European sector index}}}\ \underset{2\mathrm{x}1}{\beta} + \underset{\alpha\mathrm{x}1}{\varepsilon_i}$

4. Comparison of the results emerged from the linear regression model mentioned in the previous point and verifying that the regression analysis respects the key assumptions of the model:

i. the regressor observations are independent;

ii. $\varepsilon_i \sim N(0; V_i)$;

iii. ε_i are independent random variables, therefore there is not a serial correlation between the errors.

On the basis of the results of these analyses the index with the highest statistical robustness will be chosen. [19]

The exemplification of the diagnostic measures adopted by the Commission to make this choice follows[20]. It has been decided to proceed with graphic-type diagnostic measures; in particular, it is possible to act, with reference to the first point, through the sequence plot; this represents the regressor values in connection with time; the non-recognizability of a pattern suggests the above mentioned independence (figure 5).

As far as the second point is concerned, it is necessary to verify that errors are normally distributed with constant variance; the normality of errors can be easily diagnosed by means of the error histogram analysis, or of the qqplot; the first one represents the statistical distribution of errors and therefore allows the verification of the approximation of this diagram with the typical bell-shaped one of the normal distribution(figure 6);the second one draws a 45°line which represents the quantiles of the normal distribution; the higher the concentration of the observations identifying errors around this line the higher the normality of distribution (figure 7).

With reference to the variance constancy, it is necessary to proceed with the analysis of the residual vs fitted diagram; this diagram represents errors (residuals) related to the fitted; the presence of a clear band of observations guarantees the constancy of the variance V_i: Viceversa the determination of a possible sinusoidal oscillation as shown in figure 8 is a premonitory sign of heteroskedasticity of the variance.

The graphical analyses can be put together with numerical diagnostic tests such as the Levene or Breusch-Pagan test.

As far as the third point is concerned, we analyze the residual vs time diagram which represents the residuals in connection with time; the non-recognizability of a pattern guarantees the above mentioned independence.

[19] For a thorough explanation of the criteria used for the choice of the regressor see Greene, W.H., (1993) Econometric Analysis, Prentice Hall.

[20] For an exaustive discussion of these statistical measures see Neter, J., Kutner, M., Nachtsheim, C., Wasserman, W., (1996) Applied Linear Regression Models, The McGraw-Hill

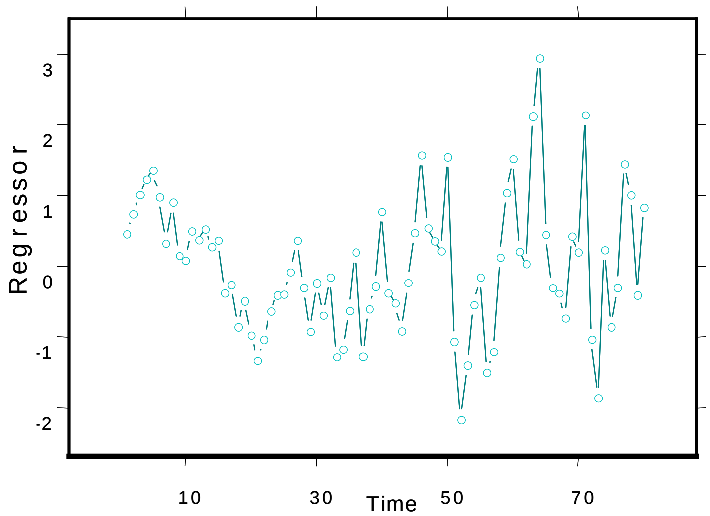

Figure 5:

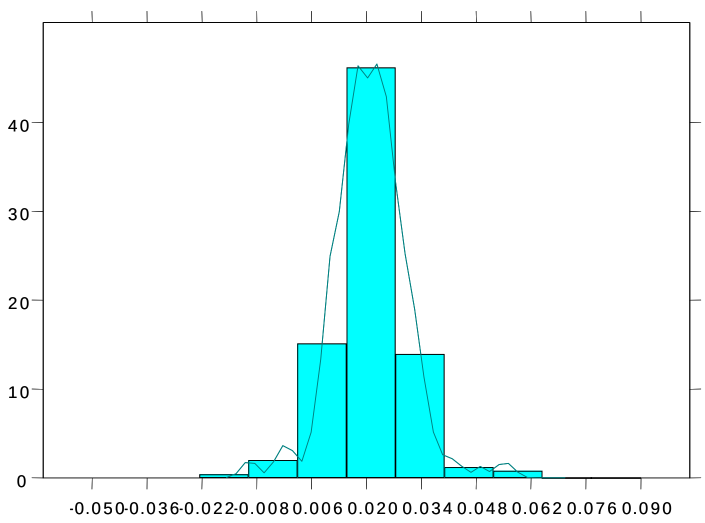

Figure 6:

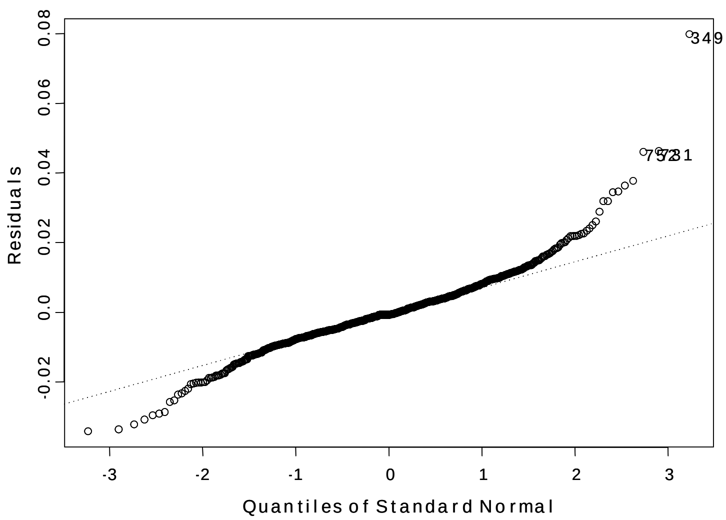

Figure 7:

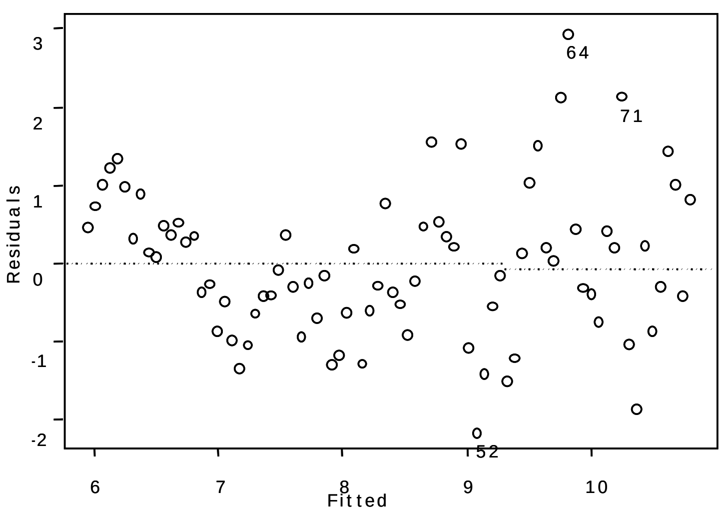

Figure 8:

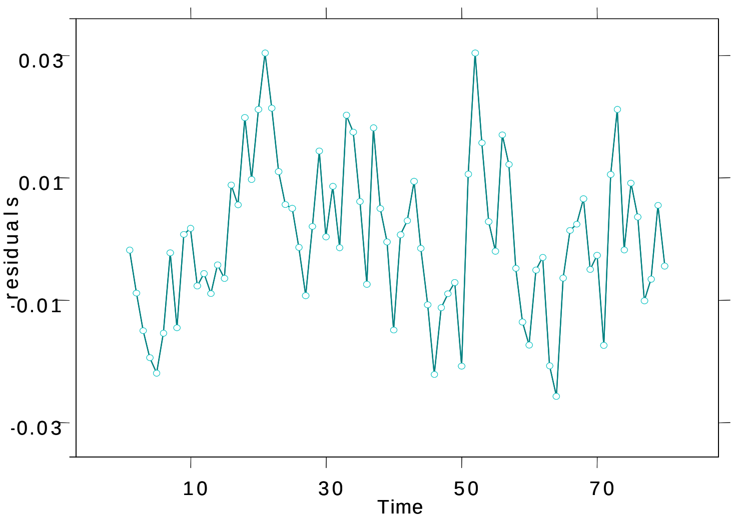

Figure 9:

5. Estimation of vector $\underset{2x1}{\beta}$ with the least squared method: $\underset{2x1}{\hat{\overrightarrow{\alpha}}}$: Moreover, in order to verify a recursive stability of parameters, there is the computation of parameters $\underset{3x2}{\hat{\beta}^0} = \begin{pmatrix} \hat{\alpha}_0^0 & \hat{\alpha}_0^1 \\ \hat{\alpha}_1^0 & \hat{\alpha}_1^1 \\ \hat{\alpha}_2^0 & \hat{\alpha}_2^1 \end{pmatrix}$ of the market model respectively for the three elements of the vector $\overrightarrow{\alpha}$. The constancy of parameters in the three periods, object of the three regression analyses, guarantees the reliability of the results of the statistical analysis. This verification can also be carried out through statistical tests such as the Chow test or graphical analyses, such as the Recursive beta diagram[21].

7. Calculation of the potential abnormal return on τ as previously seen that is:

$$\overrightarrow{e}_i = \overrightarrow{AR}_i = \overrightarrow{R}_i - \overrightarrow{R_m \hat{\alpha}}$$

The calculation of the cumulative abnormal return, the statistics construction, the hypotheses testing and the disgorgement determination are carried out as envisaged in the SEC procedure.

3.2 Problems

Both methodologies have structural weaknesses which will be explained as follows:

Companies, Inc..

[21] For an exaustive discussion of these tests see Greene, W.H., (1993) Econometric Analysis, Prentice Hall.

1. the event study applied to Insider Trading investigation determines the future trend of stock returns with a linear regression model. It is therefore based upon the assumption that these returns on a narrow interval Θ, are generated by the same linear model on the basis of parametric coefficients coming from a set of information belonging to the time window α. Hence, what has been said breaks the thesis of the weak form of market efficiency that states the impossibility of predicting the future on the basis of deterministic models, which are founded on sets of information belonging to the past, since the stock prices in the present already contain the information of the past;

2. the methodology requires a time series data set that may not be available since the stock has been recently quoted on the stock exchange;

3. the employment of a particularly long time horizon could include phenomena which have changed the company capitalization and it must be specified that the data homogenization methodologies are biased and difficult to statistically support; this happens because of the lack of a standard reference behavior of the stock market in case of regulation variation, or of extraordinary finance operations;

4. the insider trading investigation is subordinated to the determination of a reference index that is statistically meaningful as regressor and to the determination of a market portfolio model (proxy). This research is not easy for any financial market; in particular for the Italian market the presence of a high number of thin stocks hampers the implementation of the model. Moreover, the consequent concentration of exchanges on few stocks can cause regressions which are apparently statistically meaningful but are self-explicative since endogenous;

5. with reference to SEC methodology, the time horizon of 120 days is not necessarily sufficient for a time series analysis and in particular to frustrate the second term of V_i which determines, as already said, serial correlation phenomena; therefore, the regression results become invalid and statistically unreliable. Even if the usage of statistical methodology (e.g. first difference of the return) may solve the issue of autocorrelation in the period α, it's not certain that the same technique is valid in the time horizon Θ:

6. the results of the parameter time stability analysis are discriminating for the statistical investigation of the insider trading case; in other words if the parameter stability is not verified the phenomenon research can not continue without inevitable methodological problems;

7. often rumors on the stock generate spikes on the return in the period α; time reference for the parameter estimate.

In order to tackle all these problems, a new methodology, adopted by CONSOB, has been developed on the basis of the probabilistic theory which allows the discovery of the economic value of the information exploited by each insider. This procedure has been defined, as stated before, as potential probabilistic disgorgement.

3.3 Potential Probabilistic Disgorgement

3.3.1 The new approach adopted by CO.N.SO.B.

What is proposed as an alternative to the model derived from the event study analysis is a probabilistic model which simulates the stock trend in time through a stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad [1]$$

The solution of this equation is:

$$S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-s) + \sigma(W_t - W_s)} \qquad \text{where } s \leq t \qquad [2]$$

which describes in the continuum the price fluctuation of the single stock S[22] (figure 10).

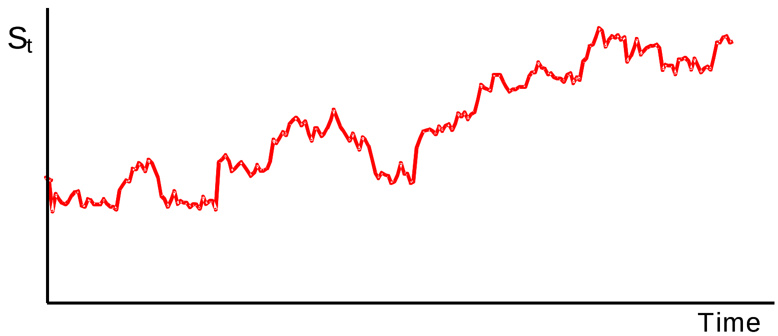

Figure 10:

With this solution it is possible to simulate the path that the stock price will follow in the future by employing the current position of the stock itself and hypothesizing a logarithmic stock return increase rate equal to $\mu - \frac{\sigma^2}{2}$ and a dispersion in this rate quantified in the parameter σ. (figure 11)

This equation benefits from the strong Markov property:

$$P(S_{T+1} = X \mid S_T; S_{T-1}; S_{T-2}; S_{T-3}; ::::::::; S_0) = P(S_{T+1} = X \mid S_T).$$

In other words, the probability that the stock price variable takes a certain value X in the future, considered the values it has assumed until the present, is equal to that conditioned only on the present. This property is then absolutely coherent with the weak form of market efficiency.

This model complies with the normal probability distribution of the logarithmic stock returns:[23]

$$\ln \frac{S_t}{S_s} \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t-s); \sigma\sqrt{(t-s)}\right)$$

This stochastic differential equation is known in probability as geometric Brownian motion and has been used in finance by Black-Scholes (1973) for their

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[22] The demonstration that [2] is the only admissable solution of [1] has been developed in appendix B.

[23] This distribution property has been demonstrated in appendix B.

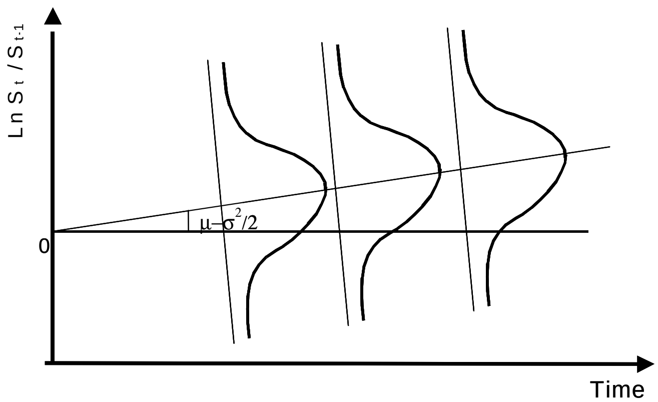

Figure 11:

well-known option pricing model[24]. Stochastic modelling has also been applied to insider trading analysis. (Grorud and Pontier, 1998).

The new methodology proposed, borrows the de¯nition of the two time horizons ® and £ from the event study analysis, but it de¯nes them in a di®erent way. ® corresponds to the period in which the insider will build his position on the stock; usually it lasts for a period that goes from 5 to 15 days before the release of the information. £ is no longer a period which contains the event but it is de¯ned by the moment in which the insider will close his position; generally the day in which the event information is given and the ¯rst or the second day after. (¯gure 12)

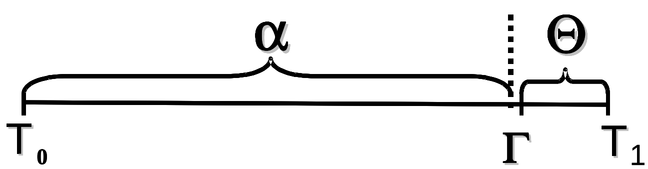

Figure 12:

The hypotheses behind these choices are that the insider:

i. cannot control what happens to the price stock dynamic before the event (i.e. the insider is price taker). This is mainly because the insider does not want to risk his trading to be recognized as insider from the market. In other words he wants to hide his insider trading strategy.

[24] A deep analysis of the main features of the equation [1] has been done in Appendix B.

ii. draws his operative strategy on the stock in the period ®: The insider, in a context of a hit and run strategy, will create a long (short) position on the stock, if the event information will have a bullish (bearish) e®ect on the price stock trend.

Therefore, the insider creates his position in the period before the information is given (i.e. the period ®), and since he knows the value of the information, it clearly emerges that he will only gain if the information will generate a higher price than the one he has incorporated in his portfolio and in a certain sense more volatility in the price than in the period ®, in which he has built his position. In other words, the insider will make a pro¯t if the information will be so price sensitive as to absorb the price oscillation that the stock has shown in the period ®. What is stated above means that the insider forecast about the stock price dynamic in the period £ is that the information will move it more than it moved in the period ®. But in terms of the stochastic methodology proposed, it means that the insider makes his pro¯t forecast based on the ¹ and ¾ determined in the period ®, in which he has created his position. Consequently the right parameters to replicate the correct price stock dynamic in the model and to quantify the insider trading disgorgement are the parameters that the insider hypothesizes and hence incorporates in his portfolio strategy. (i.e. the ¹ and ¾ in the period ®).

Moreover, as shown in ¯gure 13 every insider (i.e. Insider A or Insider B), according to his closeness to the preferential information, will have a di®erent strategy, and hence a di®erent period ®; since it will give a di®erent value to the information. Hence, this choice for the parameters should allow the model to represent the value of the information for di®erent insiders in a more realistic way.

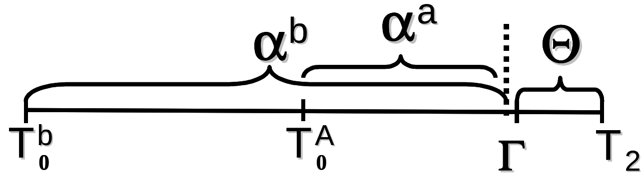

Figure 13:

On the basis of the parameters determined in the period ®, it is though possible to determine an oscillation band for the price of the stock under investigation. If there was not any event occurence, the stock price would evolve remaining in this band. This is because the insider investment strategy has been de¯ned according to the value of the information, to its price sensitivity and what's more to the fact that the information is not available to other investors. Therefore the price dynamic incorporated in the insider portfolio de¯nes the future price evolution of the stock if the information would have never been existed. (¯gure 14)

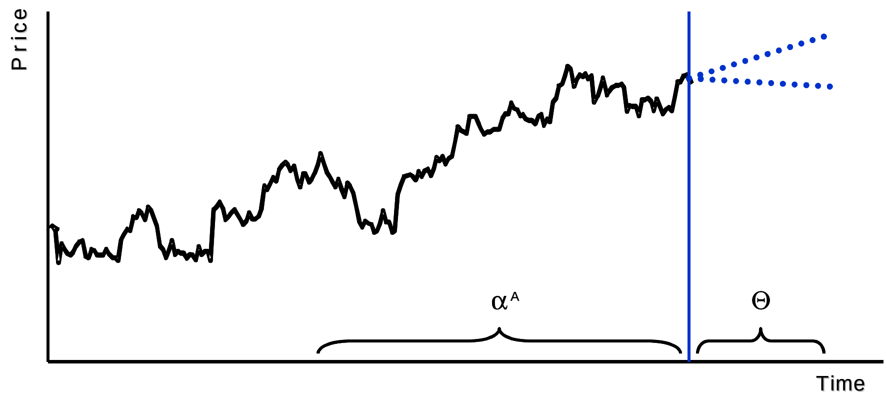

Figure 14:

The di®erence between the actual stock price after the insider information is disclosured to the market (i.e. the period £) and the band will therefore represent the value of the information the insider trader would appropriate, i.e. the digorgement. (¯gure 15)

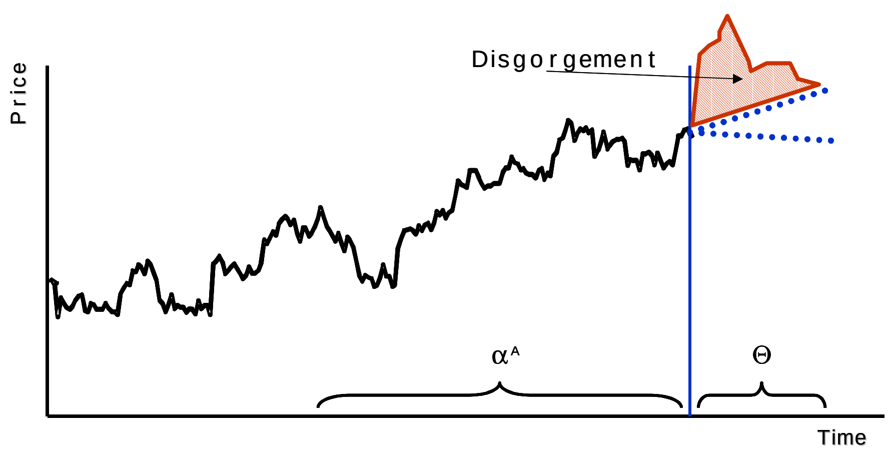

Figure 15:

Moreover, as explained before, every insider, according to his knowledge of the fraudolent information will have a di®erent investment strategy, though a di®erent stock price oscillation band and eventually a di®erent disgorgement. As in ¯gure 15 it has been shown the disgorgement for the Insider A, as in ¯gure 16 it is shown this value for the Insider B.

The model operatively develops in the following stages:

1. determination of the periods ® and £;

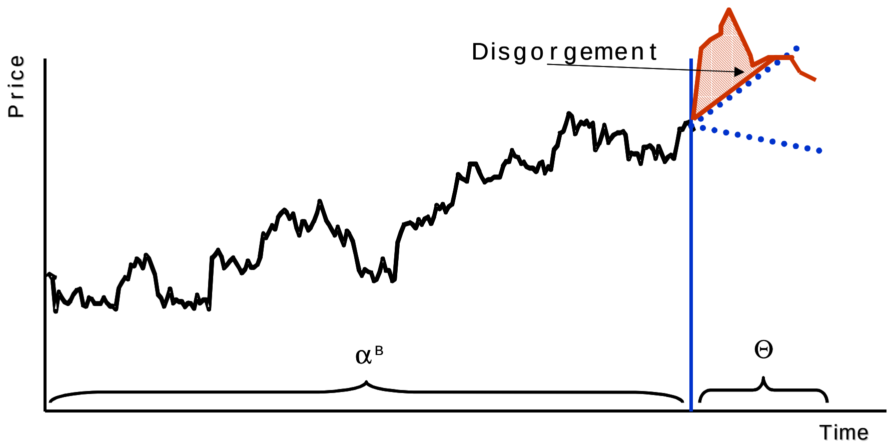

Figure 16:

2. check that there are not structural events in these periods that move the whole market hugely;[25]

3. determination of the deterministic parameters μ and σ in the period α[26];

4. individuation of an oscillation band for the prices of the stock object of study in every t[th] day of the period Θ as follows:

$$\Delta\hat{S}_t^\Theta = [S_o^\Theta e^{max}; S_o^\Theta e^{min}] \qquad [3]$$

where

$$max = \sigma z_{\frac{\gamma}{2}}\sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right)t$$

$$min = \sigma\left(-z_{\frac{\gamma}{2}}\right)\sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right)t$$

S_o is the price of the stock on the last day before the event information;

z_γ is the value of the probability density function of a standard normal random variable; by fixing γ, we identify the probability that the standard normal r.v. Z will lie between the interval $[-z_{\frac{\gamma}{2}}; z_{\frac{\gamma}{2}}]$; i.e. $P(-z_{\frac{\gamma}{2}} \leq Z \leq z_{\frac{\gamma}{2}}) = \gamma$: In other words the definition of γ determines the percentage of price evolution scenarios included in the price band. For instance $\gamma = 2{:}5\%$, means that the band will include the 97.5% of all the possible price scenarios. (figure 17)

The band determination comes out from the following hypotheses [27]:

i. $S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-s) + \sigma(W_t - W_s)}$

ii. $\ln\frac{S_t}{S_s} \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t-s); \sigma\sqrt{(t-s)}\right)$

[25] In these cases, some preliminary analyses have to be developed in order to quantify the effects of these exogenous events and try to purge the stock price trend under investigation. Although, it has to be considered that if the stock price trend is dramatically changed due to structural events, it is reasonable to hypothesize that the insider strategy is going to break down.

[26] The time reference for the computation will be daily.

[27] A comprehensive examination of the hypotheses have been shown in appendix B.

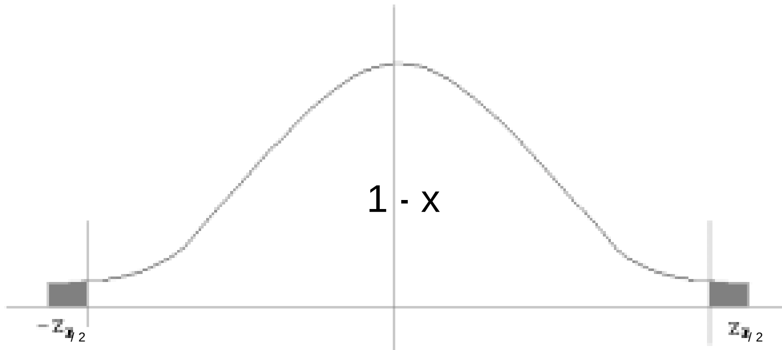

Figure 17:

iii. $P\left(S_s e^{min} \cdot S_t \cdot S_s e^{max}\right) = \{$

The choice of { determines, as said before, how many price evolution scenarios will be included at the generic time t in the price oscillation band $¢S_t^£$. Since the only reliable price to build the band for the period £ is the last price observed before the event information, de¯ned before as S_o, in the three considerations aforementioned s becomes equal to 0, and the band for the generic day t 2 £ becomes the interval expressed in [3].

5. verifying whether prices in the period £ lie or not within the oscillation band;

6. determination of the Abnormal Return as:

$$AR_t^£ = \left(S_o^£\right)^{¡1} ¢ \max\left[0; \operatorname{sign}\left(S_t^£ ¡ S_o^£ e^{max}\right) ¢ \operatorname{sign}\left(S_t^£ ¡ S_o^£ e^{min}\right)\right] ¢ \min\left[\left|S_t^£ ¡ S_o^£ e^{max}\right|; \left|S_t^£ ¡ S_o^£ e^{min}\right|\right]$$

where the sign function gives back 1 (-1) if its content is positive (negative).

7. determination of disgorgement as the quantity involved every day of the period £ in the insider trading times its correspondent abnormal return.

3.3.2 The advantages of the potential probabilistic disgorgement computation

On the basis of the explained methodological considerations and of the analysis of the model fundamental characteristics, hereafter there is the recapitulation of the advantages o®ered by the probabilistic approach adopted by CONSOB:

1. it complies with the normal distribution property of the logarithmic stock returns;

2. it allows the determination of all the possible paths of the stock under investigation under a predictive logic dynamic ;

3. it does not require a regressor since the stock path prevision only depends on the prices of the stock under analysis;

4. the definition of the parameters is extremely realistic and difficult to break down, according to the assumption that the insider will be price taker in the period in which he will create his position on the stock (i.e. period α);

5. it does not require the definition of time horizons longer than 15 days for the estimate of parameters to be employed in the analysis;[28]

6. its working is directly connected to the insider trading on the stock under investigation and, consequently to the market prices that he has incorporated through his trading strategy, it cannot be invalidated due to the fact that the company has been recently quoted;

7. it offers, through the parameters estimation procedure, a sort of customized methodology to the single subject under investigation, since the model, by construction, behaves differently according to the single insider trading strategy;

8. the computation of the disgorgement is more conservative since, instead of using the cumulative abnormal return, it is determined by directly multiplying the Abnormal return of the t^{th} day of the period Θ by the correspondent quantity of stocks involved in the insider trading; by doing so the model also considers the capability of the stock of absorbing the information;

9. the stochastic process employed benefits from the Markov property. This property makes the model absolutely coherent with the weak form of market efficiency;

10. finally, from an operative point of view:

a. it is a more intuitive approach, since it works directly on prices and not on return; moreover the reversibility between these two quantitative measures is straight forward in computation;

b. it is a faster and easier procedure in terms of implementation than the potential econometric disgorgement computation.

[28] As we have expained before, usually the insider trading strategy does not last more than 5-15 days.

4 Conclusions

The quantitative methodologies related to the analysis of insider trading are used in order to detect the phenomenon and calculate the disgorgement, which is the undue enrichment gained by the insider through the exploitation of the preferential information.

The detect phase of the insider a®ects the level of sensitivity in the market analysis carried out by the Supervisor, that is the amount of signals to put under scrutiny.

The evaluation of the disgorgement a®ects, in all the legal systems punishing the crime of insider trading, the sanction imposed against the insider and in this sense it can be considered as the linking point between ¯nancial and legal aspects.

Therefore the Supervisors envisage a lot of e®orts in the attempt to de¯ne an accurate estimate of the value of the information exploited by the insider.

This paper brings forward the di®erent methodologies developed in this ¯eld. In particular, it shows that the traditional method which computes the disgorgement as the pro¯t gained by the insider does not work, since the insider strategy is hard to reduce to a simple scheme.

Therefore, the econometric procedure developed by the SEC represents an innovative and successive attempt to produce an objective measure of the value of the information. In particular the paper shows how this methodology could be ¯tted to the di®erent features of each ¯nancial market, by developing the adaptation for the Italian one. It demonstrates that the potential econometric disgorgement computation has upgraded the procedure related to the evaluation of the pro¯t gained by the insiders but it still has some structural weaknesses such as the need of a long time series data set and of a statistically robust regressor. Moreover it has been demonstrated that these issues can completely invalidate the working of the procedure.

This work presents a new approach for the analysis of insider trading cases and the computation of the disgorgement (di®erent from the traditional event studies methods). It uses probabilistic procedures and allows the analysis of the shifts in prices of the securities in the ¯nancial markets based on the current stock price and on an analysis of all the future scenarios, giving them a suitable probability.

The potential probabilistic disgorgement computation allows the resolving of the problems a®ecting the traditional event studies methodology, such as the individuation of the market proxy portfolio, the need for a long time series data set, the temporal stability of the regression parameters and the consistency of the linearity and deterministic relation among the variables of the model.

This paper adds to the current debate on the need to regulate, enforce and supervise by using quantitative methodologies.

The use of probabilistic models in ¯nance has been corroborated in the most recent empirical analyses and the operativity of the Intermediaries presents the usage of quantitative methodology as a competitive hedge to make pro¯t and reduce ¯nancial risk.

In a world where e®ectiveness of supervision means taking enforcement action in order to protect the investors and to guarantee the e±ciency of the ¯nancial system without representing a constraint for the system itself, the use of quantitative methodology in the Enforcement process could be the solution to achieve both these targets.

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Appendix A

Glossary:

SEC: United Stated Securities and Exchange Commission.

CONSOB: Italian Securities and Exchange Commission.

Bullish(bearish) information: information that, when announced to the market, will move up (down) the stock price.

event, occurence: fact that changes the value of the company.

insider (trader): investor who accomplish an insider trading strategy.

insider trading (strategy): stock trading that is based on the exploitation of a preferential information.

disgorgement: the undue enrichment of the insider connected to the exploitation of the preferential information.

preferential inside insider fraudolent price sensitive	information:	information about a quoted company that in°uences its market price.

AR: Potential Abnormal Return.

SAR: Standardized Potential Abnormal Return.

CAR: Cumulative Abnormal Return.

Appendix B

Value of the parameters of r.v. AR

$\overrightarrow{AR_i} \stackrel{?}{\sim} N(0; V_i)$

i.e.:

i. $E(AR_i \mid R_m) \stackrel{?}{=} 0$

ii. $V_i \stackrel{?}{=} I \cdot \sigma_i^2 + E(R_m(\hat{\beta} - \beta)(\hat{\beta} - \beta)' R_m' \mid R_m)$

with reference to the mean:

$E(AR_i \mid R_m) = E(R_i - R_m \hat{\beta} \mid R_m)$

adding and subtracting $R_m \beta$; the result is:

$= E((R_i - R_m \beta) - R_m(\hat{\beta} - \beta) \mid R_m)$

$= E((R_i - R_m \beta) \mid R_m) - E(R_m(\hat{\beta} - \beta) \mid R_m)$

given the consistency properties of the estimator $\hat{\beta}$ the second term disappears.

The first term can be written as follows:

$= E(R_i \mid R_m) - E(R_m \beta \mid R_m)$

given the regression model hypothesis:

$E(AR_i \mid R_m) = 0$

q.e.d.

with reference to the variance:

$V_i = E(\underset{\times 1}{AR_i} \cdot \underset{\times 1}{AR_i'} \mid R_m) =$

rendering explicit:

$= E((R_i - R_m \hat{\beta})(R_i - R_m \hat{\beta})' \mid R_m)$

adding and subtracting in every term ε_i:

$= E((\varepsilon_i - R_i + R_m \beta + R_i - R_m \hat{\beta})(\varepsilon_i - R_i + R_m \beta + R_i - R_m \hat{\beta})' \mid R_m)$

opening brackets:

$= E((\varepsilon_i \varepsilon_i' - \varepsilon_i(\hat{\beta} - \beta)' R_m' - R_m(\hat{\beta} - \beta)\varepsilon_i - R_m(\hat{\beta} - \beta)(\hat{\beta} - \beta)' R_m' \mid R_m)$

simplifying:

$= E((\varepsilon_i \varepsilon_i' \mid R_m) + E(R_m(\hat{\beta} - \beta)(\hat{\beta} - \beta)' R_m' \mid R_m)$

simplifying:

$V_i = I \cdot \sigma_i^2 + E(R_m(\hat{\beta} - \beta)(\hat{\beta} - \beta)' R_m' \mid R_m)$

q.e.d.

Solution of equation [1]

The stochastic differential equation:

$dS_t = \mu S_t dt + \sigma S_t dW_t$ [1]

has the only admissible solution:

$S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t - s) + \sigma(W_t - W_s)}$ where $s \leq t$ [2]

First of all this equation shall have a strong solution. For this reason it is sufficient to verify that the following inequality is true:

$|\mu x - \mu y| + |\sigma x - \sigma y| \leq (|\mu| + |\sigma|) \cdot |x - y|$

The term $(|\mu| + |\sigma|)$ can be considered as a positive generic constant D and therefore the inequality is evidently true. We can conclude that this solution is unique since the Lipschitz coefficients of the differential stochastic equation are continuous.

When applying the Ito rule the result is the above mentioned solution:

$$dS = \frac{dS}{dt}dt + \frac{dS}{dW}dW_t + \frac{1}{2}\frac{d^2S}{dW^2}dt \qquad [4]$$

Then there is the computation of the derivatives:

$$S = f(t;x) = S_s e^{\left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t}$$

$$\frac{dS}{dt} = \left(\mu - \frac{\sigma^2}{2}\right) S$$

$$\frac{dS}{dW} = \sigma S$$

$$\frac{d^2S}{dW^2} = \sigma^2 S$$

Substituting in [4] the derivatives we obtain:

$$dS = \left(\mu - \frac{\sigma^2}{2}\right) Sdt + \sigma dW_t + \frac{1}{2}\sigma^2 Sdt$$

simplifying:

$$dS = \mu Sdt + \sigma SdW$$

For what has been written, we can conclude that [2] is the only admissible solution of [1].

$q.e.d.$

Brownian Motion and Geometric Brownian Motion properties

W is called standard Brownian motion. This stochastic process has the following properties:

i. $W_0 = 0$

ii. $(W_t - W_s) \sim N(0; (t - s))$i.

iii. $(W_{t_2} - W_{t_1})$ is independent from $(W_{t_3} - W_{t_2})$ where intervals $[t_3 - t_2)$ e $[t_2 - t_1)$ do not superimpose;

iv. benefit from the strong Markov Property, that is:
$P(W_{s+t} \in c \mid W_t = x; W_0 = y) = P(W_{s+t} \in c \mid W_t = x)$

v. is a continous function in time, that is with any w we have:
$\{W_t(w)\}_{t \geq 0}$ for $t \geq 0 \ast W_t$ is continous that is:
$P((w) \ast t \longrightarrow W_t(w)$ is continuous$) = 1$

It is easy to demonstrate that also $W_{t+s} - W_t = \cdot W_t$, B_s is a Brownian motion where $0 \leq s \leq 1$:

In fact:

i. $B_0 \stackrel{?}{=} 0$

$W_{t+0} - W_t \stackrel{?}{=} 0 \Rightarrow W_t - W_t = 0$

ii. $B_R - B_S \stackrel{?}{\sim} N(0; (R - S))$

$B_R - B_S = (W_{t+R} - W_t - W_{t+S} - W_t) = (W_{t+R} - W_{t+S})$
since any difference of random variables normally distributed is still normally distributed, so:
$E(W_{t+R}) = E(W_{t+S}) \Rightarrow E(W_{t+R} - W_{t+S}) = 0$
$Var(W_{t+R} - W_{t+S}) = (R + t) - (t + S) = R - S$
$\Rightarrow (W_{t+R} - W_{t+S}) \sim N(0; (R - S))$

iii $(B_{t_2} - B_{t_1})$ is independent from $(B_{t_4} - B_{t_3})$
where intervals $[t_4 - t_3)$ e $[t_2 - t_1)$ do not superimpose;
$(W_{t+t_2} - W_t - W_{t+t_1} - W_t)$ is independent from
$(W_{t+t_3} - W_t - W_{t+t_2} - W_t) \Rightarrow$
$\Rightarrow (W_{t+t_2} - W_{t+t_1})$ is independent from $(W_{t+t_3} - W_{t+t_2})$
since these intervals for hypothesis do not superimpose.

iv. B_S is continuous in t, since the difference of two random variables is still a continous random variable.
q.e.d..

For $s \to 0$ $\Delta W_t \to dW_t$ so it is possible to define in the continuum:
$dW_t \equiv \varepsilon\sqrt{dt}$ where $\varepsilon \sim N(0;1)$
and therefore for what is above written:
$dW_t \sim N(o; dt)$
dW_t is called Wiener stochastic process and can be represented as shown in figure 18.

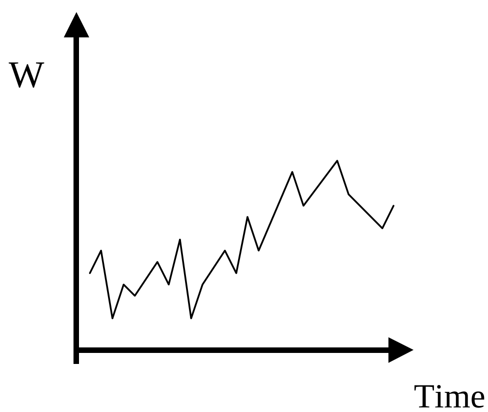

Figure 18:

Its most general version is:
$dS_t = adt + bdW_t$
where
$dS_t \sim N(a; b\sqrt{dt})$

Defining $a = \mu S_t$ and $b = \sigma S_t$ we obtain the following stochastic differential equation, known in probability as geometric Brownian motion:

$dS_t = \mu S_t dt + \sigma S_t dW_t$

Probability distribution of logarithmic stock returns

$\ln \frac{S_t}{S_{t-dt}} \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right) dt; \sigma\sqrt{dt}\right)$

by taking the natural logarithm of the solution [2] we obtain:

$\ln S_t = \ln S_s + \left(\mu - \frac{\sigma^2}{2}\right)(t - s) + \sigma(W_t - W_s)$

or $\ln S_t - \ln S_s = \left(\mu - \frac{\sigma^2}{2}\right)(t - s) + \sigma(W_t - W_s)$

In this equation we recognize the generalized Wiener process and for what stated above we can conclude that $\ln S_t - \ln S_s$; is normally distributed with parameters $a = \mu - \frac{\sigma^2}{2}$ and $b = \sigma$

$\ln S_t - \ln S_s \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t - s); \sigma\sqrt{(t - s)}\right)$

or:

$\ln \frac{S_t}{S_s} \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t - s); \sigma\sqrt{(t - s)}\right)$

and in the continuous time:

$\ln \frac{S_t}{S_{t-dt}} \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right) dt; \sigma\sqrt{dt}\right)$

q.e.d.

Normal probability distribution and stock price oscillation band

$P\left(S_{t-s}e^{min} \le S_t \le S_{t-s}e^{max}\right) = P(-z_{\{/2} \le Z \le z_{\{/2}) = \{$

where:

$Z \sim N(0;1)$ i.e. the standard normal random variable.

$max = \sigma z_{\{/2}\sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right) t$

$min = \sigma\left(-z_{\{/2}\right)\sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right) t$

by defining:

$V = \ln \frac{S_t}{S_{t-dt}}$

then by hypothesis aforementioned V is a normal random variable with parameters $\left(\mu - \frac{\sigma^2}{2}\right) dt$ and $\sigma\sqrt{dt}$: By standardizing we get:

$Z = \frac{V - \left(\mu - \frac{\sigma^2}{2}\right) dt}{\sigma\sqrt{dt}} \sim N(0;1)$

Hence:

$P(-z_{\{/2} \le Z \le z_{\{/2}) = \{$

By replacing the definition of Z:

$P\left(-z_{\{/2} \le \frac{V - \left(\mu - \frac{\sigma^2}{2}\right) dt}{\sigma\sqrt{dt}} \le z_{\{/2}\right) = \{$

Hence:

$$P\left(-z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt \le V \le z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt\right) = \alpha$$

By replacing the definition of V:

$$P\left(-z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt \le \ln\frac{S_t}{S_{t-dt}} \le z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt\right) = \alpha$$

By taking the exponential function inside the parenthesis we get:

$$P\left(e^{-z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt} \le \frac{S_t}{S_{t-dt}} \le e^{z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt}\right) = \alpha$$

By moving S_{t-dt} we get:

$$P\left(S_{t-dt}e^{-z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt} \le S_t \le S_{t-dt}e^{z_{\alpha/2}\sigma\sqrt{dt} + \left(\mu - \frac{\sigma^2}{2}\right)dt}\right) = \alpha$$

By replacing the definition of min and max we get:

$$P\left[S_{t-s}e^{min} \le S_t \le S_{t-s}e^{max}\right] = \alpha$$

q.e.d.